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AUTHOR Calabrese, Ann
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ABSTRACT

Activities for teaching mathematics in kindergarten through grade 8 are included in this guide. Both original ideas and ideas gleaned from various sources, including teachers in the district, are presented. The introduction briefly discusses the importance of developing understanding of concepts, processes, and relationships, the need for interaction and the use of all senses possible in mathematics classes; the role of evaluation; and comments on managing the class. For each activity, the objective and the procedures are included. Activities are grouped by: number and numeration system; addition, subtraction, multiplication, and division of whole numbers; divisibility tests; fractions; mixed numbers; and decimals. Also included are suggestions for teaching the basic facts, a division song, several worksheets, and some additional activities, plus a list of teacher reference materials. (MNS)

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SPRINGFIELD CITY SCHOOLS

INSTRUCTIONAL GUIDE FOR TEACHERS OF MATHEMATICS

K - 8

By Ann Calabrese
Curriculum and Instruction Facilitator

November, 1983

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INTRODUCTION

You, as a teacher, are aware that nothing in mathematics can be truly labeled as "new". Ideas and variations of ideas for teaching the numerous mathematics' concepts have been around for centuries. Many suggestions in this guide are ones I "thought" I developed and used successfully while teaching, but similar ones can be found in many books today. Using creativity, you can adjust many of these to satisfy your need. Some ideas were submitted by your fellow teachers, who will be credited with them. However, the effectiveness of any good mathematics program is a product of constantly changing interaction among the teacher, the student, and the mathematics being taught. Let the child see that mathematics is a part of every phase of life, both in material things and fantasies.

CONCEPTS

Strive always to develop understanding of concepts, processes, and relationships. Encourage children to discover solutions for themselves.

Concepts which are learned by experimenting and exploring possibilities for various solutions will be mastered more easily and will transfer to more difficult math processes.

Knowing and understanding "why" is a slower process, but hastens future learning. All of the properties in mathematics apply whether we are working with whole numbers, fractions, or integers in general mathematics, algebra, or any advanced courses.

BE PATIENT! Proceed slowly with caution, giving careful attention to process. Have students verbalize their thought processes in working specific problems. Children will achieve superior competence even though progress is retarded at first.

WHAT TO DO DURING MATHEMATICS CLASSES

Let there be teacher-pupil and pupil-pupil interaction. Talk it out! Act it out! Play it out! Explore! Discover patterns! Estimate! Round off! Test the logic of your answer! Gather statistics and graph them! Work with probability! Do group problem solving, thinking things orally -- brain storming, with no put-downs for wrong answers so a student will feel free to participate without being humiliated. Be sure that students are comfortable making mistakes. We learn from the mistakes we make. Wrong answers are keys which will help you determine a student's needs.

USE ALL SENSES POSSIBLE

SEE IT! SAY IT!
HEAR IT! SAY IT!
FEEL IT! Use manipulatives as long as needed and in all grades.

Old Chinese proverb says: "I hear and I forget
I see and I remember
I do and I understand."

Make the problems meaningful and pertain to the students' needs, giving the student an opportunity to distinguish between pertinent and extraneous information. Create a climate that will nurture an awareness of the importance of good mathematics skills to solve future as well as immediate problems.

EVALUATION

Success on tests should not be the only criterion used to measure effectiveness of your teaching. Norm referenced tests are not a measure of how well your mathematics program is doing. Teachers could teach many things which are not tested on the norm referenced test, i.e., ITBS. Criterion referenced tests, which are a direct measure of objectives taught, should be a clearer indication of what is being learned by the students.

Auditory testing eliminates student failure resulting from limited reading ability. Perform lots of oral reviews and short interviews with youngsters in problem solving.

MANAGING YOUR CLASS

Document each student's progress. This provides a systematic approach to prescriptive instruction, teacher accountability, and student accountability. Be sure students know what they are learning and the reason for being taught the objectives they are expected to learn.

Have the student ask himself questions:

What do I want to know?
What do I already know?
What am I trying to find out?

Also make students responsible for their learning.

MATHEMATICS IS FUN! LIVE IT AND YOU'LL LIKE IT! Remember, your attitude toward mathematics is learned rapidly by your students. Be positive and enthusiastic.

PREFACE

Ideas and information in this booklet have been gleaned from my 21 years of teaching elementary mathematics; courses at Central State, University of Dayton, Wright State; attendance at many state and area mathematics conferences; and, reading of educational magazines, primarily Arithmetic Teacher, published by the National Council of Teachers of Mathematics.

I strongly urge every teacher to consult Arithmetic Teacher and encourage principals to obtain a subscription for school libraries. It contains many excellent, usable ideas.

This document is incomplete. More sections are in the editing stages.

I wish to acknowledge the following:

Betty Fisher, who sat at my right side and encouraged me through anxieties in this year-long venture, read every word I wrote, and challenged me when my explanations were vague,

Ethel Baughman, who so patiently listened to my directions as she typed my copious notes and never once displayed emotions when I returned for retyping,

Teachers who sent ideas I was able to incorporate:

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Mary Mehoff, p. 133,
Ann Greer, p. 134,

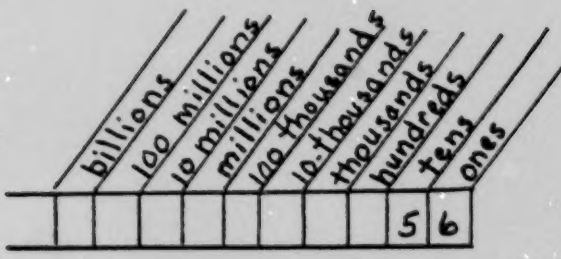
And last, but by no means least, Wilma Hall, who so carefully printed and assembled this booklet.

Ann Calabrese

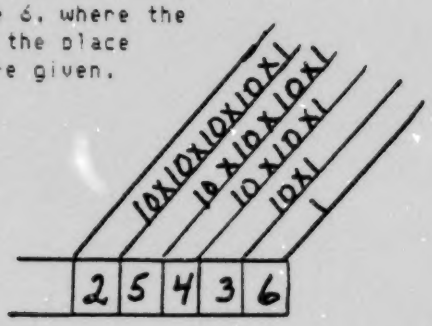
NUMBER AND NUMERATION SYSTEM: PLACE VALUE

OBJECTIVE	PROCEDURE	ACTIVITY
1. Place value -- to be reviewed and expanded upon at every grade level.	1. Concrete level of instruction will involve the use of objects which can be bundled and secured with rubber bands. Draw two columns on the board and have students draw two columns on a paper, such as shown in the activity column.	1. Every child, in the beginning, should be given a minimum of 10 objects to manipulate. Tens | Ones
2. Meaning of zero as a place holder.	2. a. Using the lined paper as a divider, have pupils place all of their objects on the left side of the line. Now write the numeral that represents the number of objects on the right side of the line. Since there are none, he should write a zero on the right side of the line, thus the development of the meaning of zero as a place holder.	2. a. Students should practice this on the board and do teacher-monitored work at their desks to be sure this concept is fully developed before moving on to anything else in mathematics. Tens | Ones 0
	b. As the pupil moves one object to the right side of the line, he/she writes the numeral one on the right side of the line to symbolize the number one. Do this with 2, 3, 4, etc., until all symbols for 0 through 9 have been learned.	Tens | Ones 1 2 3 4 5 6 7 8 9
3. Meaning of ten.	3. After developing the meaning of the ten symbols used in our Hindu-Arabic numeration system, pupils bundle 10 of their manipulative objects together and place them on the side of the line, on their paper, where tens should be (the left side of the line.)	3. Tens | Ones 1 | 0 = 10 1 | 1 = 11 1 | 2 = 12 1 | 3 = 13 etc.

OBJECTIVE	PROCEDURE	ACTIVITY
	Next, pupils write the numeral in the ten's place which represents the number of bundles of tens they now have on the left side of the line. They should write a one in the ten's place. Next step: Write the numeral on the right side of the line which represents the number of ones remaining in the one's place (illustrated in the Activity column.) Finally, write the number without the dividing line.	
4. Meaning of <u>teens</u>.	4. Give the pupils enough counters so that each has 20, and proceed, as above, to develop the meaning of numbers to 20.	
5. Meaning of <u>tens</u>.	5. Group students so there will be enough objects among them to be able to work in developing the meaning of counting and writing of numbers to 100. Continually stress that the place where a numeral is written will determine the value of the number it represents.	5. a. Use a hundred's chart to further emphasize what is being developed. b. Graduate from the use of counters to the abacus, but do so when the child feels comfortable giving up his manipulatives.
6. Use of expanded notation at the symbol level of place value instruction.	6. In the beginning, much time and effort should be spent writing on the board the value of numbers and their place in a place value grid. Ex.: 56 is 5 tens and 6 ones (5 x 10)+(6 x 1) 50 + 6 56 Then place in the grid as shown:	6. Have students make place value grid (graphing paper is ideal for this). Give them numbers to write in expanded notation.

OBJECTIVE	PROCEDURE	ACTIVITY
	 Follow this same procedure with 3 and 4 digit numbers, and increase the places of value as the child advances in the sequence and scope of the mathematics program.	Place the number in standard form on the place value grid, as set up in the procedure column on the left. It Figures, Lesson 21, SOITA, EMC Math Relationships, Lesson 6 and 9, SOITA, EMC. Measure Up, Lessons 14, 15, and 29, SOITA, EMC.
7. Necessity of a place value system.	7. In examining a large Hindu-Arabic numeral, such as 25,436, its value can more fully be understood if expanded to read (2 x 10,000) + (5 x 1,000) + (4 x 100) + (3 x 10) + (6 x 1). To continually write this number, or any large number, in expanded form would consume much time and space. If we write 10,000 as multiples of 10, it would be 10 x 10 x 10 x 10 = 10,000. 10 x 10 x 10 = 1,000 10 x 10 = 100 10 x 1 = 10 1 x 1 = 1 Study the patterns when listing multiples of tens and LET THE STUDENTS DISCOVER THAT: each number in a numeral represents a value that is 10 times the value of the number on its right.	7. Have students research the history of the Hindu-Arabic system.

OBJECTIVE	PROCEDURE	ACTIVITY
	Again, make the place value grid, and show how the place values are determined by showing the multiples of 10. Combine it with the grid made for instruction of Objective 6, where the names of the place values are given.	

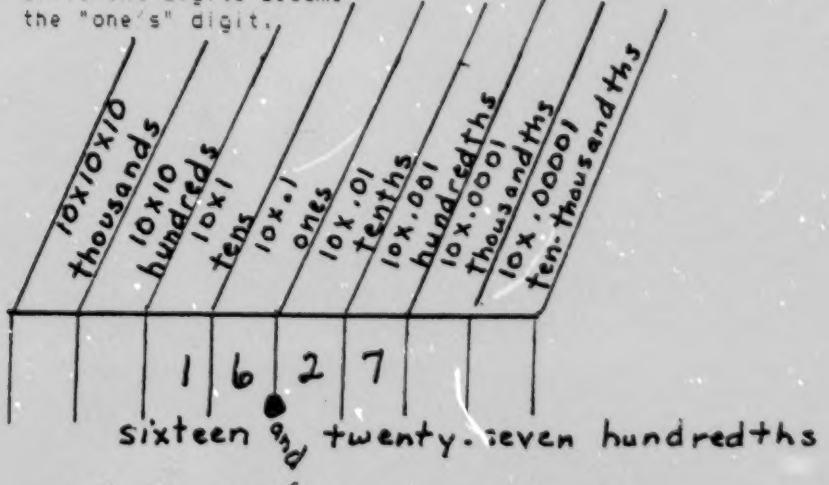


8. Powers of 10 shown with an exponent.	8. Instead of saying 10 x 10 x 10, Descartes, a French mathematician, invented a way to express repeated multiplication. He wrote the ten once and then slightly above the 10, in smaller figures, he wrote a three. Ex.: $10 \times 10 \times 10 = 10^3$ These little numbers, called exponents, will tell us how many times we multiply that number by itself. In 10^3 we say it is "10 to the third power", or the "third power of 10". Now we can show 25,436 as being $(2 \times 10^4) + (5 \times 10^3) + (4 \times 10^2) + (3 \times 10^1) + (6 \times 10^0)$. -- later written as $2(10^4) 5(10^3) + 4(10^2) + 3(10^1) + 6(10^0)$, omitting the times sign.	8. Work with multiple factors of 10 and have the student write the exponential notation. Likewise, give a number read as "power of" and let the student write it in expanded forms, using multiples of the factor. Ex.: Given $10 \times 10 \times 10 \times 10$, student writes 10^4 and, given 10^4, student writes 10,000. Set up the powers of 10 on the board and have students give the products when the value of the number indicated is found. Ex.: $10^3 = 1{,}000$ LET THE STUDENTS DISCOVER THAT: When working in powers of 10, our power number represented by the exponent will have one numeral and will be followed by as many zeros as indicated by the exponent.

OBJECTIVE	PROCEDURE	ACTIVITY
	The one's place is the zero power of 10 because it can never have the power of a ten. You can only indicate numbers whose values are less than 10 in the one's place in the base 10. Once your number becomes greater than 9 it will necessitate another place value, because it has reached a power of 10. When we write two-digit numbers we have reached a power of ten. Ex.: 16 = (1x10) + 6; 26 = (2x10) + 6 $10 = 1 \times 10^1$ $20 = 2 \times 10^1$ etc.	Ex.: 10^2 is 1 followed by 2 zeros ($10^2 = 100$) 10^3 is 1 followed by 3 zeros ($10^3 = 1{,}000$) The exponent, or numeral written indicating the power of 10 in the Hindu-Arabic system (our base 10), is always one less than the number of digits in the numeral written. 5-digit numerals minus 1 digit leaves 4-digits; thus the 4th power of a number Ex.: 25,436 is a 5-digit numeral. This would be 10 to the fourth power, or $(2\times10^4) + (5\times10^3) + (4\times10^2) + (3\times10^1) + (6\times10^0)$
9. Face value vs. place value.	9. Face value of a number never changes. A four is always a four. However, the place a number holds determines its ultimate value. In the following number, 256, the 2 has a face value of 2, but its place value is 2x100; the 5 has a face value of 5, but its place value is 5x10; the 6 has a face value of 6 and its place value is also 6 because it is 6x1. The one's place is the only place in a number where the face value and the place value are the same.	

NUMBER AND NUMERATION SYSTEM: DECIMAL SYSTEM

OBJECTIVE	PROCEDURE	ACTIVITY
10. Decimal System.	10. Explain that our base 10 system is known as the decimal system. Decimal is a derivative of the Latin word DECEM, which means 10. We also refer to our numerals in a number as digits. The Latin word for finger is digitus. Since we have ten fingers, which were used as our earliest means of counting, this perhaps accounts for the two words, digits and decimal, we use in our Hindu-Arabic numeration system.	
11. Decimal point.	11. The decimal point is a symbol used to separate a whole number from a fraction in an extension of the decimal system to tenths, hundredths, thousandths, ten thousandths, etc. Note the <u>ths</u> on the end of the names for place values on the right side of the decimal point. The decimal point also helps explain that when we multiply or divided by 10, 100, 1000 etc., different digits become the "one's" digit.	11. Do extensive board work with numbers showing how the decimal point changes in division and multiplication of multiples of 10 placing different digits in the one's place. ACTIVITIES Ex.: 546 ÷ 10 = 54.6 546 x 10 = 5460 546 ÷ 100 = 5.46 546 x 100 = 54600



OBJECTIVE	PROCEDURE	ACTIVITY
12. Multiplication of decimals by multiples of ten. (Essential concept used in division by decimals.)	12. When multiplying a decimal by multiples of 10, you may obtain your product by moving the decimal point to the right in your other factor, the same number of places as there are zeros in your multiple of 10 factor.	12. Board practice.

Ex.: Factor x Multiple of 10

.765 x 10 = 7.65 1 zero - 1 place to right
.765 x 100 = 76.5 2 zeros - 2 places to right
.765 x 1000 = 765. 3 zeros - 3 places to right
.765 x 10000 = 7650. 4 zeros - 4 places to right

In the last example, when there are not enough decimal places in the factor being multiplied by the multiple of 10 the student must annex the number of zeros needed at the end of the number to complete the process.

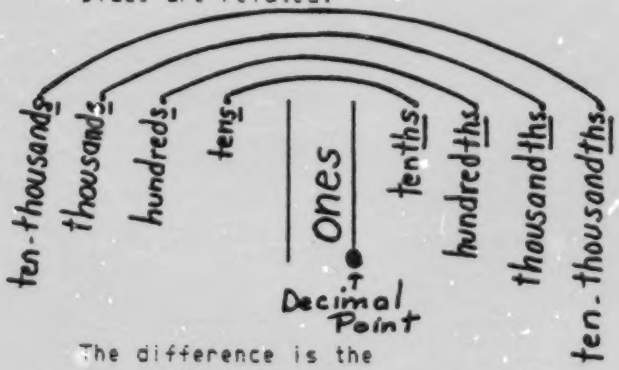
13. Division of decimals by multiples of 10.	13. When dividing a decimal by a multiple of 10, you may obtain your quotient by moving the decimal point to the left in the dividend, the same number of places as there are zeros in your multiple of 10 divisor.	

Ex.: Dividend ÷ multiple of 10

765.236 ÷ 10 = 76.5236 1 zero - 1 place to left
765.236 ÷ 100 = 7.65236 2 zeros - 2 places to left
765.236 ÷ 1000 = .765236 3 zeros - 3 places to left
765.236 ÷ 10000 = .0765236 4 zeros - 4 places to left

In the last example when there are not enough places in the dividend to complete the quotient, zeros must be prefixed before the numerals in the dividend immediately to the right of the newly placed decimal point.

OBJECTIVE	PROCEDURE	ACTIVITY
14. Decimal point in a whole number.	14. The decimal point is always to the right of the one's place in a whole number. 1. 15. 525.	14.
15. The one's place is the smallest place value for whole numbers in any base, which will be followed by a decimal point on the right to designate that fractions will follow.	15. In the decimal system (and other bases), all numbers decrease in place value as they are positioned to the right of a previously written digit. In base 10, each place in a number has a value that is 10 times the value of the number to its right.	15. Teacher-pupil interaction is best reinforcer of this concept. Board work.
16. Reading decimal fractions. a. "and"	16. a. In reading decimals the decimal point is read as "and", and separates the whole number from the decimal fraction.	16. a. Practice reading many numerals which have been placed on the blackboard. Be sure you hear the "ths" on the end of the numbers when the student reads the decimal fraction.
b. decimal's "name"	b. The name given a decimal fraction is determined by place value of the last digit to the right of the decimal point. Ex.: .215 Last digit to right of decimal is thousandths. Thus, read 215 as a whole number and name it thousandths. A student can say: (.tenths, hundredths, thousandths) which gives the decimal the name.	b. Use the gasoline pump to help understand decimals. Have students record what happens to numerals on tank when pumping gas. Have them slow the gas flow so they can see what occurs. c. When riding in a car watch moving odometer, which is based on decimal system. d. Have students make cards reading tenths, hundredths, thousandths, etc. Place numerals on the board and ask students to hold up a card which identifies

OBJECTIVE	PROCEDURE	ACTIVITY
	.2156 The last digit to right of decimal point is 10-thousandths (tenths, hundredths, thousandths, ten-thousandths.) Read 2156 just as you would a whole number and give it the name of the place value of the last digit to the right of the decimal. The one's place may also be viewed as a separation point between the decimal fraction and the whole number. There cannot be a "oneth's" place because there cannot be 10 "oneths" equaling one. 10 "oneths" would be $10 \times \frac{1}{1} = \frac{10}{1} = 10$ $10 \times \frac{0}{1} = \frac{0}{1} = 0$ Make a grid and show how the names of the place values to the right and the left of the one's place are related.  The difference is the ths on the end of the numbers to the right of the one's place as opposed to the s on the end of the number names on the left side of the one's place.	name for place value of various numerals. This provides immediate feedback for teacher and student.

OBJECTIVE	PROCEDURE	ACTIVITY
17. Zeros annexed to the right side of the decimal number.	17. Any number of zeros may be added to the right side of the decimal without changing the value of the decimal. Work through various examples relating decimals to common fractions and show the equivalence relationship which is <u>not</u> changed by annexing zeros to either the decimal fraction or the numerator and denominator of a common fraction. Example: .26000 = .2600 = .260 = .26 $\frac{26000}{100000} = \frac{2600}{10000} = \frac{260}{1000} = \frac{26}{100}$ Show how you have simplified each of the above fractions, in each step, by crossing off a zero in the numerator and the denominator, which simply means you have divided by $\frac{10}{10}$. If you crossed off three zeros in the numerator and the denominator of the first fraction, you would have been dividing by $\frac{1000}{1000}$, and you would have simplified the fraction to $\frac{26}{100}$. Go through each fraction and show how you could have arrived at $\frac{26}{100}$ earlier by dividing numerator and denominator by a greater multiple of 10 than successively dividing each numerator and denominator by 10.	17. Work many board examples so students may see the relationship between the decimal and the common fraction, such as shown with .26000 and $\frac{26000}{100000}$ Have students work out examples on the board and explain each step as they work.

OBJECTIVE	PROCEDURE	ACTIVITY
18. Zeros prefixed on the left side of the numerals, immediately to the right of the decimal point.	18. When a zero is prefixed on the left side of a decimal fraction, immediately to the right of the decimal point, it decreases the place value of the decimal fraction 10 times. Relate again the decimal to the common fraction. Ex.: $.2 = \frac{2}{10}$ Prefix a zero and you have $.02 = \frac{2}{100}$ To prove that you decreased the value of .2 ten times, you can divide $\frac{2}{10}$ by 10, or multiply $\frac{2}{100}$ by 10. Work it through this way: If you decrease the place value by 10 use $\frac{2}{10} \div 10 = \frac{2}{10} \times \frac{1}{10} = \frac{2}{100}$ or that would mean that $\frac{2}{10}$ is $10 \times \frac{2}{100} = \frac{20}{100}$, which may be simplified to $\frac{2}{10}$.	18. Work through many examples with the students so they may see how a decimal decreases the place value of existing numerals when zero or zeros are prefixed.

ADDITION OF WHOLE NUMBERS

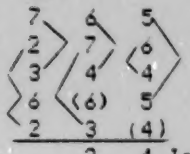
OBJECTIVE	PROCEDURE	ACTIVITY
1. Relate to idea of more. Counting is "add 1" relationship.	1. Count objects and see result when one more is added.	1. Each child keeps at least 10 objects to be used for concrete representation of number.
2. Make counting meaningful.	2. Work only with concrete.	2. Use children, books, crayons, buttons, popsicle sticks, bottle caps, beans, cuisenaire rods, etc. (to be used in first, second, and any other grade where needed.) Lesson 4, Measure Up, SOITA, EMC.
3. Meaning of addition.	3. Addition is an inclusion relation. a. Combine two or more sets of objects into a single set to show that adding is including. b. Present many examples with manipulatives so child will be able to extend counting knowledge.	3. Lesson 10, 11, Measure UP, SOITA, EMC
4. Basic Facts to 18. Sequence: Sums of 5 or less. Sums of 6 to 10. Sums of 11 to 14. Sums of 15 to 18.	4. Provide many counting experiences by using manipulatives and picture examples to relate addition facts to students. Cut large numerals from oaktag and place various combinations of the number on it. Keep them up for visual reinforcement.	4. SEE SECTION ON ADDITION AIDS AND FACTS PRACTICE, pages 110-132. a. Clap-snap-clap b. Spud Mash c. Over the mountain d. Bridge in the Canyon e. Over the Hill Subtraction f. Reverse Pyramids WHO CAN?, page 119. provides immediate feedback to teacher and student concerning fact mastery.

OBJECTIVE	PROCEDURE	ACTIVITY
5. Introduce + and - signs.	5. Introduce relationship of the two operations.	5. These symbols are like a foreign language and are a special language not easily understood by elementary children. Take time to develop simple mathematical concepts before using them.
6. Equals relation.	6. a. Show two sets of objects of same size and shape. Pupil determines number in combined sets and reads arithmetic sentence back to the teacher. Emphasize sign usage. b. Use a variety of manipulatives and have number sentences written on the board in equation form. c. Slide Rule Addition (pg. 128.) (See special section for description.)	
7. Introduce terminology.	7. Addend + Addend = Sum.	7. Use dominoes. Sections represent addends. a a a a d d d d d + d = d + d = Sum e e e e n n n n d d d d 3 + 4 = 4 + 3 = 7 Lesson 14, 18, 19, Measure Up, SOITA, EMC Use a variety of manipulatives before you present pictorial addition.

OBJECTIVE	PROCEDURE	ACTIVITY
8. Introduce equation as the name for the number sentence.	8. An equation is so named because everything on one side of the equals sign must equal everything on the other side of the equals sign. Clarify the equality which exists on each side of the equals sign. Use balance scales to show concept. There is a set in every school library.	8. (3+2)+5 = 3+ (2+5) 5 +5 = 3+ 7 10 = 10 1 + 4 =(1)+(1+1+1+1)= (1+1+1+1+1) Practice writing equations on the board, first in expanded form, then in abbreviated form.
9. Commutative property of addition.	9. Introduce commutative property. Show how the sum of two addends is the same regardless of the order of the addends. 9 + 6 = 6 + 9 a + b = b + a	9. Use concrete objects and have students make sets of different numbers. Let them discover that a set of 2 and a set of 1 will be the same number as a set of 1 and a set of 2. (++)+(+)=(+)+(++)
10. Identity Property.	10. Show how zero, when added to any other addend, does not increase the addend to which it was added.	10. Use concrete objects to demonstrate. (++)+( )=(++) Lesson 16, Measure Up, SOITA, EMC
11. Associative Property.	11. Introduce as "grouping concept". Order does not change, but groupings do.	11. (Jane + Mary) + Jess Jane + (Mary + Jess) (a+b) + c = a+(b+c) (3+2) + 5 = 3+(2+5)
12. Relate teen to ten + a single digit number.	12. Eleven and twelve are 10+ single digit numbers and are not teen numbers per se. However, thirteen is 10+3; fourteen is 10+4; fifteen is 10+5, etc.	12. Practice orally; be sure all students participate in interaction.
13. Adding Teens.	13. 14+15= (10+10) + (4+5) 14+15= 20 + 9 = 29 16+17= (10+10) + (6+7) 16+17= 20 + 13 = 20 + (10+3)= (20+10) + 3 = 30 + 3 = 33 Emphasize regrouping and renaming.	13. Do many of these orally. Mental arithmetic is slower, but produces thinkers.

OBJECTIVE	PROCEDURE	ACTIVITY
14. Relate ty to tens.	14. twenty = 2 tens thirty = 3 tens forty = 4 tens fifty = 5 tens, etc.	14. Practice renaming of multiples of ten.
15. Add tens -- no regrouping.	15. 30+60= 3 tens + 6 tens = 9 tens = 90 40+30= 4 tens + 3 tens = 7 tens = 70 or 30+60= (3+6) x 10 = 9 x 10 = 90 40+30= (4+3) x 10 = 7 x 10 = 70 Let the child discover these methods of adding tens and teens.	15. Lessons 30, 31, 32, Measure Up, SOITA, EMC
16. Add a 2-digit and 1-digit number with regrouping.	16. Be sure student understands place value and adds from right to left (ones, then tens). Ex. problem: 78 + 6 14 add the ones (1 ten + 4 ones) 70 add the tens 84 add the ones and tens Demonstrate, using counting objects to show place value and grouping.	16. Put problems on board. Have student do work orally while others monitor the work and raise hands to make corrections.
17. Add 2-digit numbers with regrouping.	17. Student must understand structure of 56 + 48 to achieve regrouping. 50 + 6 + 40 + 8 90 +14 = 90 + (10+4)= (90 + 10)+4 = 100 +4 = 104 Use counting objects to explain regrouping.	17. Lesson 30, 31, 32, Measure Up, SOITA, EMC Worksheet pg. 136, Dollar Words

OBJECTIVE	PROCEDURE	ACTIVITY
18. Addition with larger numbers (columns)	18. Have student add in groupings of tens. Ex.	18. Place Value Addition, pg. 15



2 4 In the one's column there are (5+5)+(6+4)+4 = (10+10)+4 = 24x1 = 24

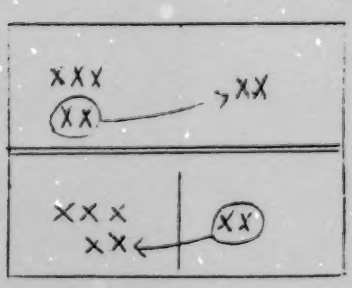
2 6 In the ten's column there are (6+4)+(7+3)+6 = (10+10)+6 = 26x10 = 260

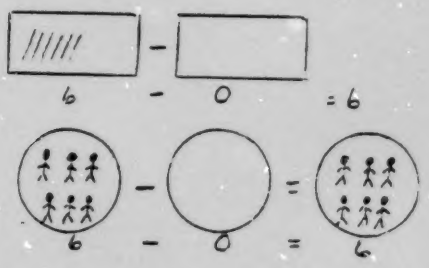
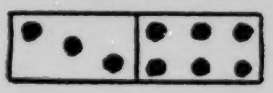
2 0 0 In the hundreds column there are (7+3)+(6+2+2) = (10+10)x100 = 20x100=2000

2 2 8 4

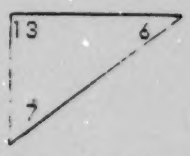
SUBTRACTION OF WHOLE NUMBERS

OBJECTIVE	PROCEDURE	ACTIVITY
1. Relate subtraction to idea of "less" or "take away."	1. a. Use concrete materials to develop the idea of a separating action. b. Each child has own collection of counters. c. Pupil sets up a certain number of objects and then removes a certain number to show how taking away, or subtraction, gives a smaller set.	1. a. Activities and Games on Measure Up, Lesson 10, SOITA, EMC c. 0 0 0 ~~0 0~~ 5 take away 2 = 3
2. Relate subtraction to idea of "more."	2. Answers the question of how many more does M have than J. a. Use one-to-one correspondence to arrive at the answer.	2. 0 0 0 0 0 M has 5 ↓ ↓ ↓ | | 0 0 0 J has 3 0 0 M has 2 more than J
3. Subtraction is the inverse of addition.	3. Use manipulatives and discover inverse relationship of subtraction and addition.	3. Give the child a paper, or cardboard, divided into two sections. On one side let the child set up a set of objects. (Use a set of 5, for example.) Have him move two into the other section of work sheet. Pupil has a visual representation of 5-2 and sees that 3 are left when 2 is taken from the set of 5. Have child move objects back together to show that by adding the two, the original set is back.

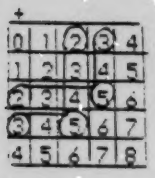


OBJECTIVE	PROCEDURE	ACTIVITY
4. Identity property of 0.	4. Any number minus, or take away, 0 does not change the identity of the set you began with. Use the separated sheet again to show that the set is unchanged.	4. 
5. Difference in subtraction is a missing addend.	5. Addend + Addend = Sum Sum - Addend = Addend a. Knowing any two numbers in either equation above, you will always be able to find the third number. A + A = S 8 + 4 = n n is found by adding n = 12 A + A = S 8 + n = 12 You know the sum and one addend. Subtract the addend from the sum and you will have the missing addend. S - A = A 12 - 8 = n 8 + 4 = 12 12 - 8 = 4 n = 4	5. a. Use students, books, and all types of objects in representing the inverse relationship. Math, No Mystery, Lesson 5, SOITA, EMC b. Use dominoes, letting the dots represent facts. Place finger over dots to be taken away and recite the fact as you are performing the operation 9 - 6 = 3 or 9 - 3 = 6 c. Use slide rule method illustrated in ADDITION AIDS pg. 116-118, 128.

OBJECTIVE	PROCEDURE	ACTIVITY
		5. d. Make triangular flash cards to show three numbers which can be related to 4 facts. Student must give the four related facts (known as families) when shown the triangular cards.

7 + 6 = 13
6 + 7 = 13
13 - 7 = 6
13 - 6 = 7

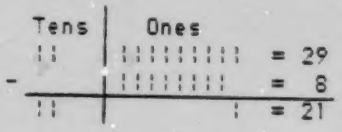


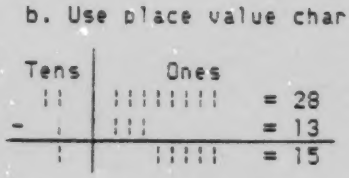
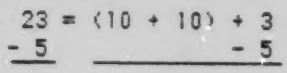
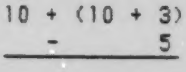
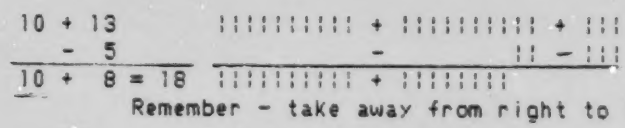
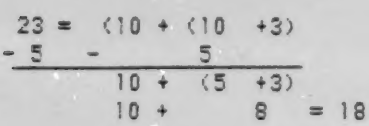
e. Use 100 addition fact chart and show inverses.

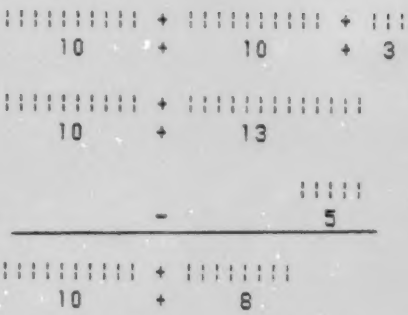


3+2=5
2+3=5
5-3=2
5-2=3

6. Comparison.	6. More, less, greater than, smaller.	
	a. Use concrete examples and let students visualize comparisons.	
7. Facts to 18.	7. See fact drills, pgs.111-119.	7.
	a. Recite entire fact as an arithmetic sentence. Don't just give the answer. (8-5=3) Eight minus five equals three.	a. Write equations on the board as teacher dictates.
		b. Races on the board, where students compete to write sentence first when given fact with one variable to find.

OBJECTIVE	PROCEDURE	ACTIVITY
		c. Drill hard on the most difficult subtraction facts: 17-9=8 13-9=4 17-8=9 13-4=9 16-7=9 13-7=6 16-9=7 13-6=7 15-9=6 13-5=8 15-6=9 13-8=5 14-9=5 12-9=3 14-5=9 12-7=5 14-8=6 12-8=4 14-6=8 11-3=8
8. Subtract a one-digit number from a two-digit number. No regrouping.	8. a. Use expanded notation. 29 = 20 + 9 -8 = - 8 20 + 1 = 21 b. Use a place value chart.  c. Always emphasize work from right to left.	8. a. Board work. Have student races doing problems. Students compete at seats and monitor board workers. b. Have students work in groups of three. Let them hand in one paper with answers upon which they all must agree after each has worked the problem. Staple their work papers to the answer sheet. If one child disagrees and will not accept the other two answers, that pupil may submit an answer on the sheet, with his/her name beside it. If he/she is right, he/she is the only one who gets credit for that problem. If wrong, he/she is only one who doesn't get credit. This gives the teacher a break. She only grades 1/3 as many papers.
9. Subtract 2-digit numbers. No regrouping.	9. Use expanded notation and place value charts as in number 8. 28 = 10 + (10 + 8) -13 = - (10 + 3) 15 10 + 0 + 5 = 15	9. Lesson 30, Measure UP, SOITA, EMC Lessons 6, 7, 8, Math, No Mystery, SOITA, EMC

OBJECTIVE	PROCEDURE	ACTIVITY
	b. Use place value chart.  Always work from right to left.	b. Use class attendance and sex distribution to make up subtraction problems. More or less (boys, girls, blondes, blue eyes, etc.)
10. Subtract 1-digit from 2-digit numbers, with regrouping.	10. Use expanded notation and renaming to show a student where he gets the 10 to add to his units.  regroup   Remember - take away from right to left.	
	Work very slowly and very methodically with this concept. Do many, many manipulatives so child fully understands the concept of regrouping.	
	Always remember to work from right to left, whether it is on the board, with manipulatives, or what. Example: 23 - 5 Could be set up as below, subtracting the 5 from the 10 borrowed and adding what is left from that 10 to the 3 in the one's place.  	See practice section, pp. 111-134.

OBJECTIVE	PROCEDURE	ACTIVITY
	Use expanded notation, with bundles of popsicle sticks, toothpicks, or whatever is available. Show 2 bundles of ten and 3 singles, from which you must take 5 singles. Let the child see you "unbundle" the one bundle of ten into ten singles. Next, show one bundle of ten, and 13 singles from which five singles are taken. Remaining are one bundle of ten and 8 singles which shows the renaming process. Repeat this many times so the child knows what renaming is. Don't proceed to 2-digit numbers until he has this concept.	
11. Subtract 2-digit numbers. with re-grouping.	11. Present as above. Example: $56 = 50 + 6 = 40 + (10+6)$ $-38 \quad -30 - 8 \quad -30 - 8$ $10 + (2+6) =$ $10 + 8 = 18$ Show 5 bundles of ten. Take one of the bundles and unbundle it to become 10 ones. When the child "borrows" the ten, he can subtract his number from the 10 and add the difference to his units place in the minuend. Explanation of above is shown in the example. It is easier to subtract 8 from 10 than it is to subtract 8 from 16.	11. Do many examples and explain as they are being done.

12. Subtraction of many-digit numerals with renaming.

PF

12. I
sp
or

Pla
impo
stan
ten
is ne

a. Ext
wit
num
Exan

renaming of 735

600 + 12
~~700~~ + 3
-200 - 80
400 + 40

OR

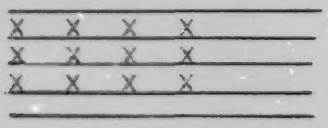
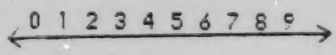
H T
6 (10+2)
~~7~~ ~~3~~ (1
- 2 8
c

on the board
align the pro

6 (10+2) (10+
- 2 - 8 - 6
4 (2 +2) (4+5
4 4 9

-23-

MULTIPLICATION OF WHOLE NUMBERS

OBJECTIVE	PROCEDURE	ACTIVITY
1. Meaning of multiplication.	1. Multiplication is a short form of repeated addition.	1. Use manipulatives and find sums of several equivalent sets. Ex.:
	a. 2 + 2 + 2 = 6	a. 3 sets of 2 00 + 00 + 00 = 000000
	b. Use patterning.	b. Skip count by twos, threes, and fives. 2, 4, 6 = 3 X 2 = 6 3, 6, 9 = 3 X 3 = 9 5, 10,15 = 3 X 5 = 15
	c. Make a large 10 by 10 square diagram on the board and use masking tape to fill in the squares so a child may visualize the facts. (The tape should be placed lightly so it can be removed easily.)  (part of the square to show 3 x 4 = 12) Remember, the first factor tells the number of the rows. The second factor tells how many are in each row. After child discovers the facts and visualizes them, build, from the discovery, the fact table the child is to memorize. Don't expect memory before understanding. Then, drill!	c. Make a large number line on the floor and have students step off facts and tell what they are doing. 

OBJECTIVE	PROCEDURE	ACTIVITY
		d. Use lessons 9 and 10 "Math No Mystery," SOITA, EMC e. Have a large number line on the board. Let one child point to the number of a set (say 2). A second child counts off another set of two and stops at four. Child will say, "2x2=4." Another child will count off 2 more and say, "3x2=6." Continue this procedure in developing facts and understanding of multiplication.
2. Property of 0.	2. Pose questions that show when zero is a factor, the product is 0. Ex.: If John has three apples and you have 0 times that amount, how many apples do you have? a. Emphasize that 0 x any number = 0. If one of the factors is 0, the product will always be 0.	2. Put many examples on the board and orally go over them to show this would be true if one of the factors is a zero even if you had a numerous amount of numbers. Examples: 0x6=0 6x6x0 = 36x0 = 0 7x2x3x0x2x4 = 0 4x6x0x5x8x7x9=0
3. Property of 1.	3. One is the identity number in multiplication. a. Discuss with the student what his identity is. (It will be confused with description.) Identity will be the name, the person he is. That name will always be who the person is. He may change it, but that will be noted on his birth certificate. The first name given will always be on his birth certificate. He may grow, change his looks, be married, or whatever, but his finger prints with which he is iden-	3. Continually ask for the property that governs the answer to a problem which has a factor of 1 or 0.

OBJECTIVE	PROCEDURE	ACTIVITY
	tified will never belong to anyone but him.	
	b. Relate this to one in multiplication and division.	
	c. Any number multiplied by one will always be the number you identified as the other factor. 8 x 1 = 8 9 x 1 = 9 0 x 1 = 0 1,234,567,890 x 1 = 1,234,567,890	
4. Commutative Property (should be able to learn in fourth grade and, at least, by fifth.	4. a. Develop meaning of commutative from the root word, commute. Ask if they know anyone who rides to and from work each day. This is commuting. The distance to and from work is the same if they travel the same route. (Children often say community or communitive instead of commutative. Be sure the word is correctly learned.)	4. a. Draw a path from home to work and another path showing work to home. Show distance is the same. HOME ——→ WORK HOME ←—— WORK
	b. Use manipulatives to show that the products of 3x2 and 2x3 are equivalent.	b. 00 000 00 equivalent to 000 00 The arrays show that: 3x2 = 2x3
	c. Explain that the first factor represents the number of rows and the second factor represents the number in each row.	c. Draw arrays on the board. Give students facts and have them draw arrays on the board which illustrate the fact given.
		d. Use sections of egg cartons to show arrays for various facts. 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 2 x 3 2 x 2 2 x 4

OBJECTIVE	PROCEDURE	ACTIVITY
5. Discovering the products of two factors.	5. a. Procedure and activities introduced in procedure and activities for Objective 1.	5. a. See Fact Practice for game of Buzz, (skip counting), pg. 119
		b. In patterning, show how the digits in the products of the 3 facts add up to some multiple of 3. 3-facts with 2-digit products are:

factor	x	factor	=	product	digits in product		multiple of 3
4	x	3	=	12	(1+2	=	3)
5	x	3	=	15	(1+5	=	6)
6	x	3	=	18	(1+8	=	9)
7	x	3	=	21	(2+1	=	3)
8	x	3	=	24	(2+4	=	6)
9	x	3	=	27	(2+7	=	9)

c. nines	c. Relate the nine's facts to multiples of 10. Using 8x9, this would be the procedure: What is 8x10? (80) Would 8x10 be greater than 8x9? (yes) Then 8x9 cannot be in the eighties, can it? (no) Will it most likely have a product in the seventies? (Do this to all of the nines) See if the students can come up with some way to memorize the nines. Lead students to discover that the number in the ten's place of the product of a nine fact will always be one less than the factor used with the nine. Ex.: 7x9 will be in the sixties. 6 plus what number equals 9? Answer: 3. 6+3=9, so 7x9 = 60+3 = 63	c. Digits in the products of the nine facts add up to a sum of 9. 1x9 = 9 2x9 = 18 (1+8=9) 3x9 = 27 (2+7=9) 4x9 = 36 (3+6=9) 5x9 = 45 (4+5=9) 6x9 = 54 (5+4=9) 7x9 = 63 (6+3=9) 8x9 = 72 (7+2=9) 9x9 = 81 (8+1=9) Lesson 10, Math No Mystery, SOITA, EMC

OBJECTIVE	PROCEDURE	ACTIVITY
	4x9 will be in the thirties. 3 plus what equals nine? (6) 4x9=36 8x9 will be in the seventies. 7+2=9. Therefore 8x9=72.	
	d. Split products. Explore the splitting of one of the factors while developing an understanding of facts. DON't RUSH INTO MEMORIZATION OF FACTS UNTIL YOU GIVE THE STUDENTS AN OPPORTUNITY TO EXPLORE!	d. Examples: 5x8 5x8 = (5x4) + (5x4) 5x8 = 20 + 20 5x8 = 40 9x8 9x8 = (5x8) + (4x8) 9x8 = 40 + 32 9x8 = 72
6. Distributive property of multiplication over addition. (See objectives 15, 16, and 17 for more in depth exploration, pp. 34,35.	6. When you work with split products you are working with the distributive property. When you multiply two digits by one digit, an example of the distributive property is: 7x56 = (7x6) + (7x50) 42 + 350 = 392 25x36 = (distributed 2 ways): a. (20x36) + (5x36) 720 + 180 = 900 b. (25x30) + (25x6) 750 + 150 = 900 Anytime you have two 2-digit numbers, you can use expanded notation with one of the factors to show how your other factor can be distributed to each place value. You will add these partial products together to obtain the product, as shown above.	6. Do lots of board work and interact with the students, using examples like those in procedure column.

OBJECTIVE	PROCEDURE	ACTIVITY
7. Memorize basic facts.	7. Review properties. Knowing properties will reduce and hasten memorization.	7. Use a hundred's facts chart and mark off all 0 facts.
	a. Property of 0.	0 x a = 0 This eliminates memorization of 19 facts.
	b. Property of 1 (identity).	b. 1 x a = a This eliminates memorization of 17 facts.
	c. Commutative property. If a student knows these properties, he will only have 36 facts to commit to memory.	c. a x b = b x a This eliminates memorization of 28 facts. If you fold a multiplication fact chart diagonally through products of the doubles, you will only have to memorize the doubles and one side of the diagonal line. 64 facts will be eliminated from those to be memorized.
	d. Memorize only a few new facts in any given lesson. Constantly review previously memorized facts.	d. See fact drills in separate section, pg. 110.
	e. Control the number of new facts added so interest and confidence will be maintained.	e. Drill a few minutes every day.
	f. Present memorization of facts as challenging and fun.	f. Assign each child a partner. Make each responsible for the other's success in memorizing facts.
	g. Monitor progress closely. Keep a daily record and progress chart. Use praise for progress and good effort.	g. Every child makes set of flash cards.

OBJECTIVE	PROCEDURE	ACTIVITY
8. Meaning of two's facts.	8. Relate doubles to addition. 2 x a = a + a 2x2 = 2+2 2x6 = 6+6 2x3 = 3+3 2x7 = 7+7 2x4 = 4+4 2x8 = 8+8 2x5 = 5+5 2x9 = 9+9 Knowing the doubles could eliminate 8 more facts from memorization.	8. Make a chart with all the facts mentioned above missing from the chart. This will show what the memorization chart looks like.
9. Multiplication algorithm.	9. Review place value which is a concern in the initial development of multiplication algorithm. a. Place algorithm on board and explain the importance of place value. This cannot be over-emphasized.	9. Use graph paper for grids or have students make their own grids. a. 32 x2 4 = 2x2 (2 ones x 2 ones 60 = 2x30 (2 ones x 3 ten product 64 is the sum of (2x2) + (2x30)
10. One-digit multiplier with renaming.	10. Try below factor "carrying" as opposed to above factor "carrying." Stress importance of place value. Ex.: 76 x2 2x6 = 12 1st partial product 2x70 =140 2nd partial product 152 product Above factor "carrying": 1 76 x2 152 2x6=12 (put down 2, carry 1) 2x7=14; 14+1 carried = 15	

OBJECTIVE	PROCEDURE	ACTIVITY
	Below factor "carrying": 76 x2 4 12 2x6=12 40 2x70=140 152	Do many examples such as: 756 x 2 1114 402 1512
11. 2-digit multiplication with re-naming.	11. Introduction shows: a. Regrouping of all partial products. b. Add all partial products to get the product of the two factors. 76 x25 5x 6= 30 5x70= 350 20x 6= 120 20x70=1400 1900	11. a. Work many of these examples and have students do them and explain the steps. Lesson 11, Math No Mystery, SOITA, EMC
12. 2-digit multiplier with "under the factors carrying."	12. Place numbers you "carry" in same area as partial products rather than above the factors. 76 x25 33 1150 420 1900 Multiplying by 5 ones: (5x6=30. Put 0 in one's place and 3 in ten's place.) (5x7tens=35 tens which equal 350. Place 5 in ten's place and 3 in hundred's place.) (Multiplication by 2 tens explained on next page.)	

OBJECTIVE	PROCEDURE	ACTIVITY

Multiplying by 2 tens: There will always be a zero in the one's place when you multiply by a number in the ten's place. Place 0 in the one's place. 2 tens x 6 ones equals 12 tens. Put 2 in the ten's place and carry the 1 into the hundred's place. Two tens x 7 tens equals 14 hundreds. Place a 4 in the hundred's place and a one in the thousand's place.

Place numbers "carried" below the factors.

In below the line carrying. you will see that a child does not have to worry about adding the carried number until he is finished.

```
   1
   3
  76
 x25
 380
152
1900
```

Example shows above the line "carrying" to be more confusing.

13. 3-digit multiplier.

(traditional and by expanded notation)

13. Explain each step thoroughly. Have child explain each step while doing board work, emphasizing why numbers are written in certain places in the algorithm.

```
  765
 x324
 3060=(4x5)+(4x60)+(4x700)=20+240+2800
15300=(20x5)+(20x60)+(20x700)=100+1200+14000
229500=(300x5)+(300x60)+(300x700)=1500+18000+210000
247860 (sum of all three partial products)
```

OBJECTIVE	PROCEDURE	ACTIVITY

"Over the factors carrying"
(The traditional way of teaching multiplication. Note complicated carrying.)

```
   11
   11
   22
   765
  324
 3060
1530
2295
247860
```

a. 1st partial product: (ones x ones) + (ones x tens) + (ones + hundreds)

2nd partial product: (tens x ones) + (tens x tens) + (tens x hundreds)

3rd partial product: (hundreds x ones) + (hundreds x tens) + (hundreds x hundreds)

Add all partial products together.

14. 3-digit multiplier.

"Under the factors carrying."

A new way!

"Under the factor carrying" eliminates adding in the number he "carries" while he is still in the process of multiplying.

Example:

```
  765
 x324
 222
  840
 111
  420
 211
  185
247860
```

Using this method, the student never loses track of his facts. He can always check his facts, e.g., 2 = 20 (0) 3 = 36 (6)

OBJECTIVE	PROCEDURE	ACTIVITY
	Show that when he is "carrying", in the area of partial products, he is actually writing a fact.	
15. Distributive property of multiplication over addition.	15. Pretend student cannot multiply by any number greater than 5 and needed to know 5x9. Rename 9 into 5+4. Now your problem is 5 x (5+4).	

Draw arrays to show 5x5 and 5x4.

5x5	+	5x4
00000		0000
00000		0000
00000		0000
00000		0000
00000		0000
25	+	20 = 45

As you build arrays with manipulatives, close the gap between the two arrays and you have a 5x9 array.

000000000
000000000
000000000
000000000
000000000
5x9 = 45

16. Distributive property of multiplication over addition where a single digit factor is distributed to a 2-digit factor.	16. Multiply 7x37. Rename 37 into (30+7). Now you have 7x(30+7). Emphasize that any time you have a multiplication problem which has one factor with more than one digit in it, it must be solved by using the distributive property of multiplication over addition.

$$\begin{array}{r} 37 \\ \times 7 \\ \hline 7\times 7 = 49 \\ 7\times 30 = 210 \\ \hline 49 + 210 = 259 \end{array}$$

OBJECTIVE	PROCEDURE	ACTIVITY

7x (30+7) means that you multiply everything in the parenthesis by 7. Now your problem is (7x30)+(7x7)

(7x30) = 210
(7x7) = 49
(7x37) = 259

"Over the factors carrying":

```
  4
 37
 x7
259
```

"Under the factors carrying":

```
 37
 x7
 24
  19
259
```

17. Distributive property of multiplication over addition where a 2-digit factor is distributed to a 2-digit factor.

17. Using the problem 27x25:

a. First let's rename the 27 into (20+7). Now, distribute 25 to 20 and 25 to 7, e.g., (25x20) + (25x7) = 500 + 175 = 675.

b. Rename the 25 into (20+5): Now distribute 27 to 20 and 27 to 5. e.g., (27x20) + (27x5) = 540+135 = 675.

c. Rename both factors and multiply:

25 = 20 + 5
27 = (x20 + 7

(7x20) + (7x5) = first partial product
(20x20)+(20x5) = second partial product
(20x20)+(20x12)+ (7x5) = product
400 + 240 + 35 = 675

OBJECTIVE	PROCEDURE	ACTIVITY

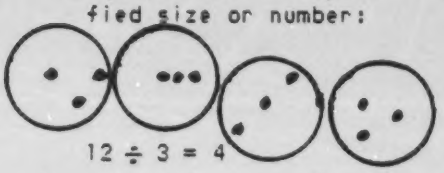
d. Same problem with "Over the factors carrying."

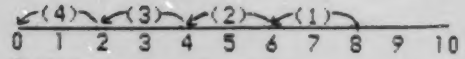
$$\begin{array}{r} {\scriptstyle 1} \\ 3 \\ 25 \\ \times 27 \\ \hline 175 \\ 50 \\ \hline 675 \end{array}$$

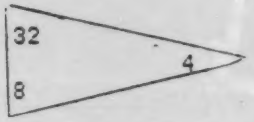
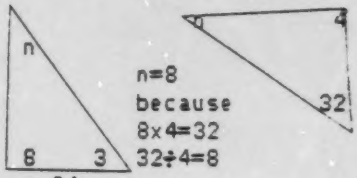
e. Same problem with "under the factors carrying."

$$\begin{array}{r} 27 \\ \times 25 \\ \hline {\scriptstyle 13}\backslash\ 05 \\ {\scriptstyle 1}\backslash \\ 44 \\ \hline 675 \end{array}$$

DIVISION OF WHOLE NUMBERS

OBJECTIVE	PROCEDURE	ACTIVITY
1. Meaning of division.	1. a. Sharing. Share things common in children's everyday experiences with candy, toys, etc.	1. Use concrete objects in the room and share them. a. 1) 16 popsicle sticks shared evenly with 4 children (16÷4). 2) 8 baseball cards shared evenly with 2 children (8÷2). 3) Use a large set and have children divide it into equal sets of a specified size or number:  12 ÷ 3 = 4 Use Math No Mystery Lessons 9, 12, 13, SDITA, EMC.
	b. Repeated subtraction. Demonstrate repeated subtraction and its relation to division. Make sure concept is understood before giving problems for practice.	b. 1) There are 24 sticks of gum. I will give 4 to the first people who arrive. How many people will get 4 sticks of gum? 24 -4 (1) 20 -4 (2) 16 -4 (3) 12 -4 (4) 8 -4 (5) 4 -4 (6) 0 24÷4=6 Six people get 4 sticks of gum.

OBJECTIVE	PROCEDURE	ACTIVITY
		2) Use number line to show repeated subtraction.  8÷2=4 (4 jumps of 2 in each jump)
c. Inverse of multiplication. In multiplication we multiply a factor x a factor to obtain a product. In division we are searching for a missing factor. If we take a product and divide it by a factor we know, we will obtain the factor we do not know.		c. F x F = P (factor x factor = product) P ÷ F = F (product ÷ factor = factor) Lesson 9, Math No Mystery, SOITA, EMC
d. Have students work out many equations and talk the problem through as they work it on the board. (THIS SHOULD FOLLOW MANY EXAMPLES USING MANIPULATIVES. A CHILD MUST BE GIVEN MANY CONCRETE EXAMPLES AND UNDERSTAND THE CONCEPT BEFORE BEING GIVEN PENCIL AND PAPER WORK WITH SYMBOLS. IF A CHILD IS TO FULLY UNDERSTAND DIVISION, HE/SHE MUST FULLY UNDERSTAND MULTIPLICATION AND VICE VERSA.)		d. F x F = P 8 x 3 = 24 3 x 8 = 24 P ÷ F = F 24 ÷ 3 = 8 24 ÷ 8 = 3 24 ÷ n = 8 (P ÷ F = F) 24 ÷ 8 = n (P ÷ F = F) 24 ÷ 8 = 3 ; therefore 24 ÷ 3 = 8 n ÷ 8 = 3 (P ÷ F = F) 8 x 3 = 24 therefore F x F = P 24 ÷ 8 = 3
e. Set a work center where children may practice finding missing factors or products in the division equation.		e. Find missing factors or products if you know that F x F = P and P ÷ F = F Ex.: If 8x7=n then n÷7=8 If nx7=28 then 28÷7=n If 4xn=48 then n=48÷4

OBJECTIVE	PROCEDURE	ACTIVITY
2. Division facts.	2. a. Form families of facts from a knowledge of the inverse relationships of multiplication and division. (Zero cannot be used as one of the factors because dividing by zero is undefined.) See Obj. 15, pg. 54.	2. a. Use triangular flash cards. Child states 4 complete facts formed by the three numbers. (Cannot be done with zero as a factor.) 
	b. Illustrate division facts.	b. Make board drawings partitioning sets. Use concrete objects.
	c. Make triangular flash cards with only two numbers on them, and have children recite entire facts, supplying the missing number.	c.  n=8 because 8x4=32 32÷4=8 n=24 because 24÷8=3 8x3=24 24÷3=8 3x8=24
	d. See section on fact drills, pg.110.	d. Drill for memorization of facts, only after the child understands the division concept.
3. Divisor	3. a. Divisor tells the number of objects in a set -- or -- the number of sets into which you wish your objects to be divided. $\text{Divisor})\overline{\text{Dividend}}$ Quotient	
	b. Divisor is a factor in a multiplication problem. $\text{Factor})\overline{\text{Product}}$ Factor	
4. Quotient	4. a. Quotient can name the number of sets formed from a given number of items -- or -- name how many can be in each set if your divisor	

OBJECTIVE	PROCEDURE	ACTIVITY

specifies the number of sets to be obtained from the partitioning.

6 (number in each set)
5)30 (number of things to be divided)
(number of sets wanted as shown by divisor.)

6 (number of sets)
5) 30 (number of things to be divided)
(number to be placed in each set as shown by divisor.)

b. Quotient is the missing factor in a multiplication problem.

quotient
30 ÷ 6 = 5 because 5 (the missing factor) x 6=30

5. Dividend

5. a. Dividend names the number of things in a large set which are to be divided into smaller sets.

Quotient
Divisor) Dividend

5. Do extensive board work, with much teacher-pupil interaction.

Have the pupils name the parts of a division problem when first beginning the operation.

b. Dividend is the product of a multiplication problem.

Factor
Factor) Product

c. When writing a division problem in horizontal form, the dividend is written first in the equation.

Dividend ÷ Divisor = Quotient

Product ÷ Factor = Factor

OBJECTIVE	PROCEDURE	ACTIVITY
6. Estimating quotient in division by use of rounding dividend and divisor.	6. Provide a variety of estimating activities. Ex. $45\overline{)2854}$ Estimating first digit of quotient and the number of digits in the quotient. Round off 45 to 50 and 2854 to 3000. $50\overline{)3000}$ 50 will not go into the dividend 1000 times because 50x1000=50,000. 50 will not go into the dividend 100 times because 50x100=5000. 50 will go into 300 six times since 5 tens goes into 30 tens 6 times. In 3000 the 300 is 300 tens. Therefore, the first numeral in the quotient will be directly above the 0 which is in the ten's place of the dividend. By rounding and estimating you have found 6 to be a trial quotient for our problem. Just remember! It could be too large, or it could be too small. But, try it! $\begin{array}{r} 6 \\ 45\overline{)2854} \\ \underline{270} \\ 154 \end{array}$ Six is the first numeral in your quotient and it is in the ten's place. This will tell you that your quotient will be a two-digit quotient in the sixties. Next step will be to see how many times 50 will go into 150 and use that number as your trial number for the one's place. 50 should go into 150 as many times as 5 goes into 15. So, try 3. 3x45=135 which	6. a. Give students problems to work out method of rounding and estimating and have them walk it through step by step. b. It Figures, Lesson 22, SOITA, EMC, Estimating When Dividing c. Math, No Mystery, Lessons 4, 12, 13, SOITA, EMC

OBJECTIVE	PROCEDURE	ACTIVITY

is less than 154.
154-135=18. The remainder is less than the divisor 45. Thus your division is completed.

```
    63
45)2854
  270
   154
   135
    18 R.
```

REMEMBER! TRIAL QUOTIENTS WON'T ALWAYS BE AS ACCURATE AS THE EXAMPLE ABOVE!

7. Several methods which can be used to teach division.

7. a. Division by expanded notation:
Ex.: 156 ÷ 2

(100÷2) + (50÷2) + (6÷2)=
50 + 25 + 3 =
50 + (20+5) + 3 =
Use associative property:
(50 + 20) + (5+3)=
70 + 8 = 78

7. a. Math, No Mystery, Lesson 9, SDITA, EMC.

This is a drawn out procedure, but the time spent doing it will pay off a hundredfold.

Give simple problems such as the one example, first, and gradually increase to to thousands and ten thousands.

b. Division of 2 digit numerals by one digit.
Ex. 1:

```
      11     R7   Step 4
    8)95
Step 1  80  | 10   partial
        15  |    + quotients
Step 2   8  |  1
Step 3   7  | 11   Step 4
```

b. It Figures, Lesson 17, SDITA, EMC.

Step by step explanation below:

Step 1. How many sets of 8 are there in 90? (Ans.10) Place a 10 on the right side of the dividing line. Place the product of 8x10 on the left side of the dividing line, even with the partial quotient 10. Subtract 80 from 90 and you have a remainder of 15.

OBJECTIVE PROCEDURE ACTIVITY

Step 2. How many sets of 8 are there in 15? (Ans. 1) Under the 15 place the product of 8x1 and to the right of the line, directly across from the product 8, place a 1 under the one's place in the number 10.

Step 3. Subtract 8 from 15 and you have a remainder of 7.

Check your remainder to see if you can make another set of 8. Seven is smaller than 8, so 7 becomes your remainder.

Step 4. Add the two partial quotients and place the complete quotient above the dividend in their respective place value. (11 R 7)

Ex. 2:
Same problem as Ex. 1, except placement of partial quotients is above the dividend.

```
                (add 1st and 2nd
                partial quotients
                to get
  11 R 7        complete quotient)
    1 >         (second partial quotient)
   10 >         (first partial quotient)
 8) 95
   80  =  (10x8) - record 10 above
   15                 dividend
    8  =  (1x8)  - record 1 above
    7  Remainder       dividend
```

Ex. 3: Is the short form, which the student is striving for:

```
      1
   8) 95
Step 1  8
       15
```

(1 and 8 in 10's place is using our knowledge of place value) 10x8=80 (1 remainder in the ten's place means that you can not get another set of 10 eights out of 90 and you have the 1 ten and 5 ones left.

OBJECTIVE	PROCEDURE	ACTIVITY
	Step 2 $8\overline{)15}$	15 now becomes the dividend. How many sets of 8 can you get out of 15? (Ans. 1)
	Step 3 $8\overline{)15}$ $\underline{8}$ 7	Multiply 1x8 and subtract the product from 15. The remainder is 7. It is smaller than the divisor, which means your division is completed.
	STEP 4 $\begin{array}{r}11\ R\ 7\\ 8\overline{)95}\end{array}$	Place 1 in the quotient in the one's place, to the right of the one in the ten's place. Record your remainder to the right of the quotient.
8. Remainder.	8. Remainder is what is left, or remains, when a large set has been divided into smaller sets of a specified number and is too small to form another set of the number specified by the divisor.	8. Divide the class into groups of various sizes to show the meaning of remainder. Ex.: 24 students will make 4 sets of 5 and the four remaining will not be enough to form another set of 5. 4 remains.
9. Five ways to arrive at a complete quotient (2-digit by 1-digit divisor). p.q. refers to partial quotient; c.q. refers to complete quotient.	9. a. $\begin{array}{r}13\ R\ 1\\ 6\overline{)79}\\ \underline{60}\\ 19\\ \underline{18}\\ R.1\end{array}$	10 (x6) p.q. is 10 3 (x6) p.q. is 3 13 (x6) c.q. is 13R1 (place quotient above dividend.)
	b. $\begin{array}{r}13\ R\ 1\\ 6\overline{)79}\\ \underline{30}\\ 49\\ \underline{30}\\ 19\\ \underline{18}\\ R.1\end{array}$	5 (x6) p.q. is 5 5 (x6) p.q. is 5 3 (x6) p.q. is 3 13 (x6) c.q. is 13R1 (place quotient above dividend.)

OBJECTIVE	PROCEDURE	ACTIVITY
	c. 13 R 1 6)79 42 — 7 (x6) p.q. is 7 37 36 — 6 (x6) p.q. is 6 R.1 — 13 (x6) c.q. is 13R1 (place quotient above dividend.)	
	d. 13 R 1 6)79 6 — 10 (x6) p.q. is 10 19 18 — 3 (x6) p.q. is 3 R.1 — 13 (x6) c.q. is 13R1 Utilizes knowledge of place value where 6 is used for 60 and placed in the ten's place.	
10. Several ways to find the quotient of a 4-digit number.	10. Ex.: a. 346 R3 8)2771 1600 — 200(x8) p.q.is 200 1171 800 — 100(x8) p.q.is 100 371 320 — 40(x8) p.q.is 40 51 48 — 6(x8) p.q.is 6 R.3 — 346(x8) c.q.is 346 R 3 This is a multiple subtraction method which does not lock you into a specific number in your quotient.	10. Permit the student to experiment to see how many ways he/she can work the problem. Although these methods are long, and are time consuming, they teach a lot of mathematics. To name a few concepts that he must know and use: Place Value Distributive Property Division is a short form of subtraction.

OBJECTIVE	PROCEDURE	ACTIVITY

b.

```
346 R 3  c.q.
  6 p.q.
 40 p.q.   add partial
100 p.q.   quotients
100 p.q.   up.
100 p.q.
8)2771
 800     100 (x 8) p.q. is 100
1971
 800     100 (x 8) p.q. is 100
1171
 800     100 (x 8) p.q. is 100
 371
 320      40 (x 8) p.q. is  40
  51
  48       6 (x 8) p.q. is   6
 R.3     346 (x 8) c.q. is 346 R 3
```

Each time the child finds a partial quotient, it is written above the dividend. After exhausting all of the multiples of 8 which can be extracted from 2771, add them to find the sum of the sets of 8 which can be obtained from the division of 2771. The sum is the complete quotient.

11. Short form of division which you wish for the student to be able to master.

11. Walk the child through the following steps as many times as is necessary to spell out for him exactly what each step means:

```
   346 R 3
8) 2771
 24       300(x8) p.q. is 300
 371
 320       40(x8) p.q. is  40
  51
  48        6(x8) p.q. is   6
 R.3      346(x8) c.q. is 346R3
```

11. Concepts needed:
Place value
Use of zero s a place holder.

OBJECTIVE PROCEDURE ACTIVITY

Explanation of the short form above:

a. How many thousands of sets of 8 can you get out of 2000? Ans: none.

b. How many hundreds of set of 8 can you get out of 2700? Ans: 300.

By placing a 3 in the hundred's place of the quotient you have written 300. The product of 300 x 8 is 2400.

Two in the 1000's place and 4 in the 100's place would be 2400, which you place under 2771.

c. Subtract 2400 from 2771 and you have a remainder of 371 (zeros can be inserted to hold place value.)

d. How many tens of sets of 8 can you get from 371? Ans. You will get about as many tens of sets of 8 as there are eights in 37 tens -- which is 40. Therefore there are 40 sets of 8 in 371. You show this by placing a 4 in the quotient in the ten's place.

e. The product of 8x40 is 320, which you subtract from 371. You have a remainder of 51. Now you see how many sets of 8 you can get out of 51. Since you can get 6 sets of 8, you place a 6 in the one's place. Subtract 6x8 from 51 and you have a remainder of 3.

OBJECTIVE	PROCEDURE	ACTIVITY
12. Division by 2-digit divisor.	12. Examples and explanations of steps: Example A: $28\overline{)76}\ 2$ 56 20 Thinking through: Round off 28 to 30. Round off 76 to 80. There are 2 thirties in 80, so there should be about 2 twenty-eights in 76. Use 2 as a trial quotient. $2 \times 28 = 56$ $76 - 56 = 20$ (check remainder as shown) Is the remainder smaller than the divisor? If so, it is known as the remainder. Ans. 2 R 20. Example B: $17\overline{)96}\ 5$ 85 11 Round off 17 to 20. Round off 96 to 100. There are 5 twenties in 100, so there should be about 5 seventeens in 96. Use 5 as a trial quotient. $5 \times 17 = 85$ $96 - 85 = 11$ (check remainder as shown) Is the remainder smaller than the divisor? If so, it is the remainder. Ans. 5 R 11. Example C: $18\overline{)92}\ 4$ 72 20 Round off 18 to 20. Round off 92 to 90.	12. Always walk students through each step and let them walk you through each step.

OBJECTIVE	PROCEDURE	ACTIVITY

There are 4 twenties in 90, so there should be about 4 eighteens in 72. Use 4 as a trial quotient.

4x18=72
92-72=20 (check remainder as shown)

Is the remainder smaller than the divisor? (no) This lets us know that another set of 18 can be made from the 20. So, our trial quotient is not large enough.

Remainder is large enough to make one more set of 18. Increase quotient from 4 to 5.

```
   5
18)92
   90   5x18=90
    2   92-90=2
```

Is the remainder smaller than the divisor? If so, the trial quotient now is correct and the remainder is 2. Ans. 5 R 2.

Note: The word trial means exactly what it says. It is a quotient to try. It may sometimes be too small and sometimes may be too large.

13. Division of 3-digits by 2-digit divisors.

13. Thinking through each step for problem:

```
      28
33)956
   660
   296
   264
   R32
```

13. SLOW PROCESS, YOU SAY? CORRECT, BUT -- take time to work with the child and walk her/him through many examples.

As you walk the pupil through, gradually

OBJECTIVE	PROCEDURE	ACTIVITY
	a. Estimate answer first. Round off 33 to 30. Round off 956 to 960. 960÷30= a little more than 30. (Answer will be somewhere close to 30 -- maybe more, maybe less.)	eliminate parts of your explanation for the student to fill in, until he is giving the entire explanation of steps by himself.
	b. Now, student knows that the quotient is a 2-digit number. This indicates that the ten's place of that digit will be directly above the ten's place in the dividend. (That will be directly above the 5 in the dividend.)	IT WORKS! I'VE TRIED IT!
	c. Mental arithmetic should come into the picture here. Can the child mentally multiply 3x33 and see that 3 is too big? (If not, let's teach how to mentally multiply.)	
	d. Some will not be able to see 3 as being too large. So let them discover that 3 is too large. Be sure pupil checks remainder each time he/she subtracts to see if it is smaller than the divisor.	
	e. After trying 2 and placing it in the ten's place, always have pupil verbalize that the two is actually 20 because of its place value. So he/she is multiplying 20x33 -- not 2 x 33. The answer then is 660, not 66.	
	f. When subtracting 660 from 956, one gets 296. Check the remainder, after placing a finger over the one's place, and see if that part of the re-	

mainder is less than the divisor. (If so, proceed. If not, increase the number in the quotient and work through above process again.)

g. Using the remainder 296, round off 33 to 30. Round off 296 to 300.
$300 \div 30 = 10$

h. One cannot place a 10 in a place value slot because 10 itself is a 2-place-value number. So, use 9 as a trial quotient.

i. Multiplying 9x33 obtains a product of 297. It is obvious one cannot subtract 297 from 296, so the trial quotient number 9 was too large. Drop back to 8 as the trial number.

j. Multiplying 8x33=264.

k. Subtract 264 from 296. The remainder is 32.

l. Check the remainder to see if it is less than divisor. If so, it is the remainder. If not, then use another trial quotient.

m. From step i, if 9x33 was one too many, it should help you see that the remainder will be 1 less than the divisor.

OBJECTIVE	PROCEDURE	ACTIVITY
14. Division by 3-digit divisor.	14. A. Example of division by subtraction method: 10+5+3= 18 R 39 157)2865 1570 10 (x157) 1295 p.q. is 10 785 5 (x157) p.q. is 5 510 471 3 (x157) p.q. is 3 R 39 18 x 157 c.q. is 18 R 39 In the method shown above, the student does not have to have the exact trial number for the quotient. By subtracting different quantities of the specified size of the set (noted by the divisor) and then adding up the number of sets subtracted altogether, is not a bad way to arrive at the complete quotient. It may be more meaningful to the slower learner. Naturally, we hope for the shorter method -- but don't discount other ways to work a problem. There is no one way for computation in the basic operations.	14. A. MENTAL MATHEMATICS IS IMPORTANT! SOME OF IT SHOULD BE DONE DAILY. IT IS CERTAINLY HELPFUL IN THE OPERATION OF DIVISION.
	B. Short form of division: 18 R 39 157) 2865 1570 10(x157) p.q. is 10 1295 1256 8(x157) p.q. is 8 R 39 18(x157) c.q. is 18 R 39	B. When you work with large numbers and round off, your range of possible solutions becomes greater and requires more mental mathematics, more analytical thinking, and more estimating.

OBJECTIVE	PROCEDURE	ACTIVITY
	Steps:	
	a. Ask the pupil to first see if he/she can take 150 into 2800 twenty times. If the student sees that 2x15=30 and 20x150=3000, he/she will be able to estimate that any number in the twenties will be too large. This will be a starting point toward the solution -- the answer is in the teens. This will necessitate that a one be placed in the ten's place of the quotient, directly above the ten's place in the dividend. A one in the ten's place means 10.	a. Finding a solution set is similar to Goldilocks' search for a suitable bed -- "not too big, not too small but just right."
	b. Multiply 10x157. Product is 1570.	
	c. Subtract 1570 from 2865. The difference is 1295.	
	d. Place a finger over the one's place in the difference to see if the remainder of 129 is less than the divisor. It is, so 1 in the ten's place of the quotient is the correct number. (You already knew it was a one in the ten's place from working through step a above.)	
	e. Finding the trial quotient number for the one's place becomes a trial-and-error method, with reasoning. It must be less than 10 because there can only be a single digit in any place value position.	

OBJECTIVE	PROCEDURE	ACTIVITY
	Trial-and-error: 9x157=1413 (this is too large) 8x157=1256 (this is smaller than 1295, so it becomes the number to place in the one's place of the quotient.) Subtract 1256 from 1295 and you have a remainder of 39, which is less than the divisor. Your problem is now completed.	
15. Division by zero is undefined, or meaningless.	15. By using many examples of families of facts formed by inverse operations of multiplication and division, one can prove that you can only obtain 3 facts, or true statements, when one of the factors is 0. Sample proof: $8 \times 9 = 72$ F1 x F2 = P $9 \times 8 = 72$ F2 x F1 = P $72 \div 9 = 8$ P ÷ F2 = F1 $72 \div 8 = 9$ P ÷ F1 = F2 $8 \times 0 = 0$ F1 x F2 = P $0 \times 8 = 0$ F2 x F1 = P $0 \div 8 = 0$ P ÷ F1 = F2 then $0 \div 0$ should be 8 but $0 \div 0 \neq 8$ P ÷ F2 ≠ F1	15. Have students work out families of facts where zero is one of the factors. Also, work out families where zero is not a factor to show you will always obtain 4 related facts, as opposed to 3 when one factor is zero. Note: Make sure child understands the final reasoning. Impress it on pupils.

DIVISIBILITY TESTS

1. A number is divisible:

	Examples:
a. <u>by 2</u>, if the unit's place is divisible by 2, e.g., 2, 4, 6, 8 or if the number is a multiple of 10.	a. 2)764 2)850 382 425
b. <u>by 3</u>, if the sum of the digits is divisible by 3.	b. 483 The sum of the digits of 4+8+3=15. 15 is divisible by 3; therefore 483 is divisible by 3.
c. <u>by 4</u>, if the last two digits of a number are divisible by 4.	c. Using the number 316, the last 2 digits are 16. Sixteen is divisible by 4; therefore, 316 is divisible by 4.
d. <u>by 5</u>, if the units place is 5 or 0.	d. 725 ends in 5. 725 divided by 5 = 145. 650 ÷ 5 = 130.
e. <u>by 6</u>, if the last two digits are divisible by 2 and the sum of the digits is divisible by 3.	e. 624. The last two digits are 24. 24 is divisible by 2. And -- the sum of 6+2+4 = 12. 12 is divisible by 3. Both conditions are true, therefore. 624 is divisible by 6.
f. <u>by 7</u>, if, after you strike out the last digit in your numeral (4) -- double the digit you struck out (4x2=8) and subtract that answer from the remaining number 57-8=49, your answer is divisible by 7, then, the original number is divisible by 7 (49÷7=7).	f. 57~~4~~ cast out 4 double 4 = 8 57 - 8 = 49 49 is divisible by 7; so, 574 is divisible by 7.
g. <u>by 11</u>, if, after you strike out the last digit in the numeral (1) and subtract the numeral you struck out from the remaining number (89-1=88), and arrive at a numeral that is divisible by 11.	g. 89~~1~~ struck last digit. <u>-1</u> subtract it from 89 that remains 88 is divisible by 11 and so is 891.
h. <u>by 11</u>, (for use if numbers are very large), if, after you add all digits in odd numbered places, right to left, then -- add all digits in even numbered places, right to left, and after subtracting the smaller from the larger sum, obtain a difference	h. 9,270,563 3 + 5 + 7 + 9 = 24 sum of odd 6 + 0 + 2 = <u>8</u> sum of even 16 difference 16 is not a multiple of 11 and the difference between odd and even numbers was not 0; there-

of 0, 11, or a multiple of 11.

Examples:

fore, 9,270,563 is not divisible by 11.

932,382
8 + 2 + 9 = 19 sum of odd
2 + 3 + 3 = -8 sum of even
11 difference
Since the difference is 11, 932,382 is divisible by 11.

i. by 13, if, after you strike out the numeral in the one's place -- then multiply that number by 9 -- then subtract that product from the number that was left -- and your difference is divisible by 13.

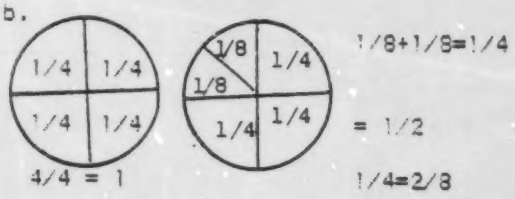
i. 1183 1 1 8 3 strike last digit.
3x9=27
118 - 27 = 91
91 ÷ 13 = 7
91 is divisible by 13, so 1183 is divisible by 13.

j. by 25, if the last two digits in a number are divisible by 25. Therefore, the last two digits of your number will either be 00, 25, 50, or 75.

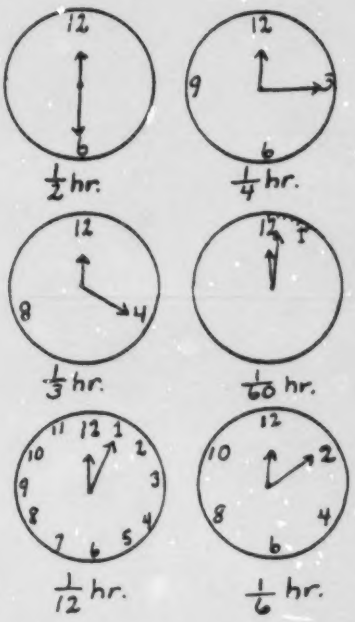
j. 200 ÷ 25 = 8
425 ÷ 25 = 17
350 ÷ 25 = 14
175 ÷ 25 = 7

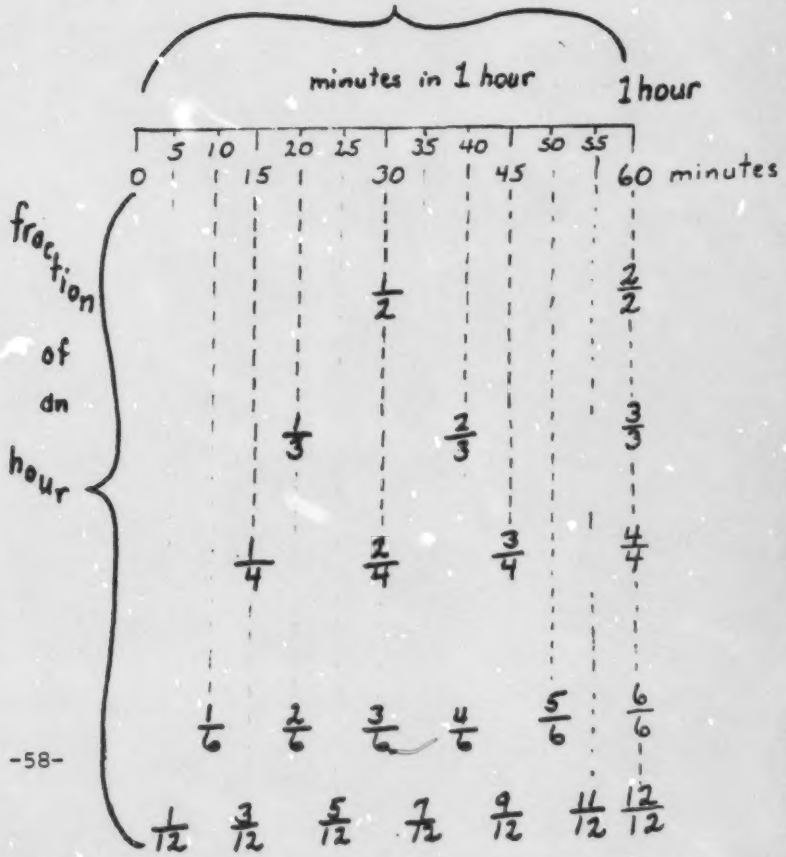
Computation, A Changing Perspective,
Ohio Department of Education,
Columbus, Ohio, 1982

FRACTIONS

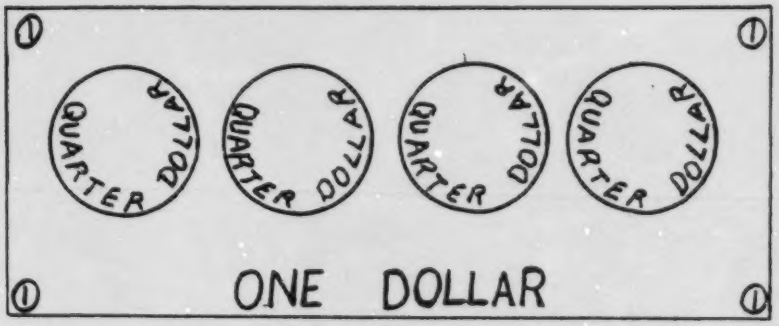
OBJECTIVE	PROCEDURE	ACTIVITY
1. Meaning of a fraction as an equal part of a whole: 1/2, 1/3, 1/4. Introduce other fractions to students as the course of study dictates. 1/6, 2/3, 3/4, etc.	1. Choose ideas appropriate for ability of your students.	
	a. Use concrete objects and divide into equal parts. Show how parts must be equal before you can name them as fractions.	1. a. Divide something enjoyable and practical, such as candy, cake, fruit, brownies, or pizza. Children will usually volunteer something.
	b. Place diagrams of sectioned shapes on the board and have students orally name parts specified.	b. 
	c. Use concrete objects to help child determine the written symbol for a fraction. (Students are ideal manipulatives.)	c. Ex.: Use a set of 6 bottlecaps. 1 is 1/6 of the set 1/6 says one out of 6
	d. Use egg cartons. Cut them into sections representing various fractional parts of the dozen. Color the sections. Use a whole egg carton and place your colored section inside the uncolored whole carton, so he/she may see the relationship of the section to the whole.	d. Student names fractional part that is colored. 1/3 4/12 = 1/3
	e. Use large number line depicting the number one. Divide that one into equal parts and label the parts. Student will discover equivalences of fractions by seeing that one part may have several different names, determined by the way the whole is divided.	e. See model enclosed from Fraction Bars Kit, by Albert B. Bennett, Jr. and Patricia S. Davidson (Creative Publications, Palo Alto, Calif.), pg. 134. Film: MP 1594 T, What Are Fractions and How They Are Used, EMC Lesson 26, 27, Measure Up, SOITA, EMC

OBJECTIVE	PROCEDURE	ACTIVITY
	f. Use the clock as a model. Relate: 1/2 of the clock to 1/2 hour. 1/4 of the clock to quarter 'til or quarter after the hour. Divide the clock into 20-minute sections and show that 20 minutes is 1/3 of the hour; 10 minutes, 1/6 of the hour; 5 minutes, 1/12 of the hour; and 1 minute, 1/60 of the hour.	f.





OBJECTIVE	PROCEDURE	ACTIVITY
	g. show relationship of quarter to 1/4 by showing a quarter as 1/4 of a dollar.	g.

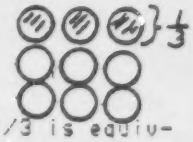
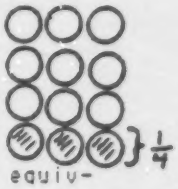


	h. Use cuisenaire rods.	h. What Is A Fraction?, SFS, 4347T, EMC. Math No Mystery, Lesson 17, SCITA, EMC Parts of Things (Film) AA0249 E, EMC
2. Meaning of Denominator.	2. a. The denominator is the numeral that is written below the line of the fraction symbol. $\frac{x}{5}$ denominator	
	b. The denominator indicates the number of equal parts into which a whole has been divided. $\frac{x}{5}$ says the whole is divided into 5 equal parts.	

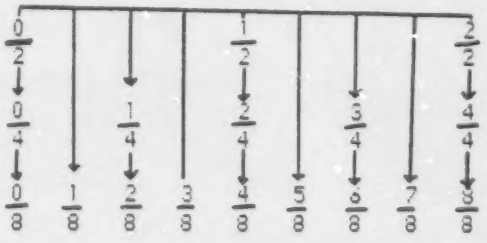
OBJECTIVE	PROCEDURE	ACTIVITY
	c. The denominator is the name of the fraction. $\frac{x}{2}$ halves $\frac{x}{3}$ thirds $\frac{x}{22}$ twenty seconds (seconds when a 2-digit number ends in 2.) All the rest of the fractional names ending in 4, 5, 6, 7, 8, 9, and multiples of 10 end in ths or ieths, e.g., fourths, fifths, twentieths.	
3. Meaning of Numerator.	3. a. The numerator is the numeral that is written above the line of the fraction symbol. $\frac{4}{4}$ numerator	
	b. The numerator names the number of equal parts of the whole with which we are working. $\frac{4}{5}$ 4 out of 5 equal parts	
	c. The numerator is always read as a whole number.	c. Practice orally reading the names for fractional symbols. Math No Mystery, Lessons 16, 18. SOITA. EMC
4. A fraction is a division problem.	4. a. Every fraction is a division problem. Read all fractions from top (numerator) to bottom (denominator). 4/5 is read as 4 divided by 5.	

OBJECTIVE	PROCEDURE	ACTIVITY
	b. The numerator is the dividend. $5\overline{)(4)}$ is 4/5	
	c. The denominator is the divisor. $4 \div (5) = 4/5$	
5. A fraction is the quotient -- or the answer to a division problem.	5. a. Have students orally read a fraction as a division problem, and -- give the same fraction as the quotient. 4/5 means 4 divided by 5 and the answer to that division problem is 4/5. $4 \div 5 = 4/5$	
	b. Use improper fractions to help the student see that the fraction is also the quotient for the division problem it represents. The fraction 6/2 means 6 divided by 2. 6/2 is an answer, which we rename as 3. (1) (1) (1) 2/2 + 2/2 + 2/2 = 6/2 = 3	
6. A fraction may represent a part of a set.	6. a. Teacher questioning procedure: 1) How many checkers do we have altogether? Ans. (11) 2) How many of the checkers are red? (7) 3) What part of the whole set is red? (7 out of 11)	6. a. Use many examples. One might be a set of checkers (7 red: 4 black)

OBJECTIVE	PROCEDURE	ACTIVITY
	4) What fraction represents this answer? (7/11)	
	b. (Ex. 2)	b. Ex.: Set of 4 bottle caps and 9 bottles.
	1) How many objects in the set? (13)	
	2) How many objects are bottle caps? (4)	
	3) What part of the set is bottle caps? (4 out of 13 or 4/13)	
	4) What part of the set is bottles? (9 out of 13 or 9/13)	MP 1595T, Fractions II Some Operations With Fractions. EMC
7. Fractions show ratio.	7. a. Ratio is the comparison of one number with another. It shows a relationship between 2 numbers, or 2 measurements. The number you are comparing to, or comparing with, is the denominator of the fraction. Ex. Ratio is written as 2:4. As a fraction you write it 2/4. 2 is being compared with, or to 4. Or, it answers the question of "What part of 4 is 2?"	7. a. Give the students many of these statements and have them write the fraction on the board.
	b. In measurement you always compare the same units. Feet to feet Inches to inches Meters to meters, etc.	b. Math Relationship, Lesson 1, SOITA, EMC
	c. Rule of thumb: The number following compare to, compare with, or part of is the denominator of your ratio.	

OBJECTIVE	PROCEDURE	ACTIVITY
	Compare 2 with 6 -- 2/6 Compare 5 to 7 -- 5/7 What part of 4 is 3? -- 3/4	
8. Ratio and Proportion.	8. a. Equivalent ratios are proportions. If I set up a ratio 2:3, it becomes a problem in proportion when I set up an equivalent fraction to that ratio. Ex.: 2:3::4:6 -- read as, 2 is to 3 as 4 is to 6. or can be written 2/3 = 4/6. Ex.: 1:2::2:4 or 1/2 = 2/4 If you are given 1/2 of a pie, you would receive the same proportion if the pie were cut into fourths and you received 2/4 of the whole pie.	8. a. 2/3 = 4/6 Proportions can be found and taught by same method used to find equivalent fractions. Draw pictures showing proportions. Make arrays which show proportions.  1/3 is equivalent to 3/9.  1/4 is equivalent to 3/12
	b. If a proportion is being set up, one must be sure that the numerators of the fractions represent the same things -- as is also true of the denominators. (If one numerator is inches, the other numerator of your equivalent fraction must be inches; if denominator is feet other denominator must also be feet.)	b. Some problems to show procedure: 1) If 2 cows give 4 gallons of milk, and all cows give the same amount of milk, how many gallons of milk will 6 cows give? $\frac{2 \text{ (cows)}}{4 \text{ (gals)}} = \frac{6 \text{ (cows)}}{12 \text{ (gals)}}$ 2 : 4 :: 6 : 12 In such a problem, I can make cows or gallons numerators or denominators and still be able to solve the problem.

OBJECTIVE	PROCEDURE	ACTIVITY
		2) If there are 25 pennies in 1 quarter, how many pennies are there in 5 quarters? $\frac{1}{25}$ (quarter) $\frac{5}{125}$ (quarters) (pennies) = (pennies) 1 : 25 :: 5 : 125 Or, if 1 quarter has 25 pennies in it, how many quarters can you exchange for 125 pennies? $\frac{1}{25} = \frac{x}{125}$ = $\frac{1}{25} = \frac{5}{125}$ quarters to quarters pennies to pennies
c. If a problem sets up a comparison in different measurements, you must change so both are the same units of measurement before you can make a comparison. Ex.: If there are 12 inches in a foot, how many inches are in 1 yard? You must change your yard to feet before you can set up a proportion. You cannot compare $\frac{\text{inches}}{\text{feet}} = \frac{\text{inches}}{\text{yards}}$ It must be $\frac{\text{inches}}{\text{feet}} = \frac{\text{inches}}{\text{feet}}$ $\frac{12}{1} = \frac{36}{3}$ inches to inches feet to feet 12 : 1 :: 36 : 3	c. Give problems orally where changes must be made and have students set up the proportion on the board.	

OBJECTIVE	PROCEDURE	ACTIVITY
9. Extremes and Means.	9. a. If the equation for the proportion is written as a fraction, the numerator of the first fraction you write and the denominator of the second fraction you write are known as the <u>extremes</u>. Or, if your proportion is written 1:2::2:4, they are still the first and last numbers written. They are at the <u>extreme</u> ends of the symbol.	9. a. Show these on the board. extremes (1) : 2 : : 2 : (4)
	b. The two numbers in the middle of the symbol 1:2::2:4 are the <u>means</u>. If written as fraction equation, denominator of the first fraction and numerator of the second fraction you write are the <u>means</u>.	b. 2 : (7) : : (6) : 21
10. Proof that your fractions are equivalent and your proportion is correct.	10. a. If the product of the extremes is equal to the product of the means, then the fractions are equivalent to each other and your proportion is correct.	10. a. means 2 : 7 : : 6 : 21 extremes 2x21=7x6
	b. Draw number lines to show ratio and proportion.	$\frac{1}{2} = \frac{2}{4}$ 1x4=2x2
11. Equivalent fractions.	11. a. Use number line to show equivalent fractions.	11. a. 

OBJECTIVE	PROCEDURE	ACTIVITY
	b. Place one set of multiples above another set of multiples. Result will be equivalent fractions.	b. $\frac{2}{3} = \frac{4}{6} = \frac{6}{9} = \frac{8}{12} = \frac{10}{15} = \frac{12}{18}$ (multiples of 2) (multiples of 3)
12. Simplifying fractions.	12. In simplifying a fraction, the student is finding an equivalent fraction with a smaller denominator. Divide fraction by the equivalence of one, using the greatest common factor of the numerator and denominator of the fraction you wish to simplify.	12. Do board work to illustrate: $\frac{4}{8} \div \frac{4}{4} = 1$ $\frac{8}{4} \div \frac{4}{4} = 2$
13. Finding greatest common factor (GCF) of two numbers.	13. a. The GCF (Greatest Common Factor) is the largest number we can find which will divide into both numerator and denominator of a fraction without leaving a remainder.	13. a. Board work, with step-by-step explanation - first by teacher - then by pupil, as is shown below.
	b. Find pairs of factors for numerator and denominator of fraction you wish to simplify.	b. Ex.: Simplify 12/42: Factors of 12: 3, 2, 1, (12) 12, 6, 4 Factors of 42: 6, 3, 2, 1, (42) 42, 21, 14, 7
	c. Find the largest factor common to both 12 and 42. This will be the GCF.	c. Both have 1, 2, 3 as factors -- 6 is the Greatest Common Factor.

OBJECTIVE	PROCEDURE	ACTIVITY
14. Use of identity number in division, renamed as a fraction.	14. a. After finding the GCF of the numerator and the denominator of a fraction (see #13), divide the numerator and the denominator by the GCF. In essence, you are dividing by the equivalence of 1 and will obtain a fraction equivalent to the one you are simplifying. The GCF of 12 and 42 is 6. By dividing 12 and 42 both by 6 (the GCF of 12 and 42), you will obtain the simplest form of $\frac{12}{42}$. $\frac{6}{6}$ is the equivalence of 1. so, we are dividing by the equivalence of the identity number in division. $\frac{12}{42} \div \frac{6}{6} = \frac{2}{7}$ (see objectives 19 and 20.)	14. Find the GCF of many pairs of numbers and proceed as above.
15. Finding GCF by Euclid's Maze.	15. a. Divide the smaller number into the larger number. b. The remainder becomes the new divisor; and, the divisor of the first step in the problem becomes the new dividend for the second step. The remainder of each step becomes the divisor for the succeeding step. The divisor of each step becomes the dividend for each succeeding step. c. Repeat the process used above until the remainder obtained is zero. d. The last divisor used is the GCF of the two numbers.	15. Work through examples and explain.

OBJECTIVE	PROCEDURE	ACTIVITY

Example: Find the GCF of 453 and 876.

1st step
$$453\overline{)876}\ 1$$

2nd step
$$R\ 423\overline{)453}\ 1$$

3rd step
$$R\ 30\overline{)423}\ 14$$

4th step
$$3\overline{)30}\ 10$$
$$\underline{30}$$

Dividend of first step is the larger of the two numbers. Divisor of first step becomes dividend of 2nd step. Divisor of 2nd step becomes dividend of 3rd step. Divisor of 3rd step becomes dividend of 4th step.

3 is your last divisor. Therefore 3 is the GCF.

Divisor of last step is GCF.

Example: Find the GCF of 27 and 87.

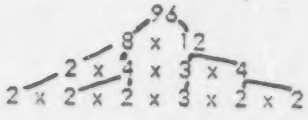
$$27\overline{)87}\ 3$$
$$\underline{81}$$
$$6\overline{)27}\ 4$$
$$\underline{24}$$
$$3\overline{)6}\ 2$$
$$\underline{6}$$

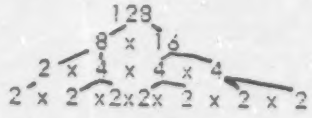
3 is last divisor. Therefore, 3 is the GCF.

16. Simplifying a fraction by prime factoring.

16. Find the set of prime factors for two numbers involved in a simplification.

a. Example: Find the prime factors of 96 and 128 by use of factor trees.





b. set up fraction with sets of primes for each numerator and denominator.

$$\frac{96}{128} = \frac{2 \times 2 \times 2 \times 2 \times 2 \times 3}{2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2}$$

16. Pair off students and have them do board work, factoring numbers into sets of primes. Teacher supervises work closely.

OBJECTIVE PROCEDURE ACTIVITY

Next step is to cross out all primes in the numerator that are in the denominator, matching one to one.

$$\frac{96}{128} = \frac{\cancel{2} \times \cancel{2} \times \cancel{2} \times \cancel{2} \times \cancel{2} \times 3}{\cancel{2} \times \cancel{2} \times \cancel{2} \times \cancel{2} \times \cancel{2} \times 2 \times 2}$$

c. What you have left is $\frac{3}{2 \times 2} = \frac{3}{4}$

d. Your simplified fraction is the primes which are left. You had 3 in the numerator and 2x2 in the denominator. Therefore, $\frac{96}{128} = \frac{3}{4}$

e. If you wish to know what the GCF is, it is the product of the primes you struck out in the numerator (or the denominator since they are the same.)

They were 2x2x2x2x2, which equals 32.

$$\frac{96 \div 32}{128 \div 32} = \frac{3}{4}$$

f. What if all prime factors were cancelled out in the numerator because of a one-to-one matching with primes in the denominator?

Ex.: $\frac{64}{128} = \frac{\cancel{2} \times 2 \times \cancel{2} \times 2 \times 2 \times \cancel{2}}{\cancel{2} \times \cancel{2} \times \cancel{2} \times \cancel{2} \times \cancel{2} \times \cancel{2} \times 2}$

You must consider that you divided by 64, the product of cancelled primes, in the numerator and know your answer is not $\frac{0}{2}$, but

is $\frac{64}{128} \div \frac{64}{64} = \frac{1}{2}$

OBJECTIVE	PROCEDURE	ACTIVITY

So, once you get your set of primes, you can always put <u>one</u> as a factor of the numerator and denominator because a factor of one does not change the product. Then you cancel out all common primes. Since 1 is not a prime, it will never be cancelled and you'll always have at least one number in your numerator.

$$\frac{64}{128} = \frac{\cancel{2} \times \cancel{2} \times \cancel{2} \times \cancel{2} \times \cancel{2} \times \cancel{2} \times 1}{\cancel{2} \times \cancel{2} \times \cancel{2} \times \cancel{2} \times \cancel{2} \times \cancel{2} \times 2 \times 1} = \frac{1}{2}$$

17. Finding the GCF by working with factors of the numerator and denominator.

17. Set up the factors of both numbers concerned in the simplification.

17. Much board work and oral reasoning.

Find the GCF of 12 and 36.

Factors of 12 -- 2, 3, 4, 6, (12)
Factors of 36 -- 2, 3, 4, 6, 9, (12)
GCF of the two numbers is 12.

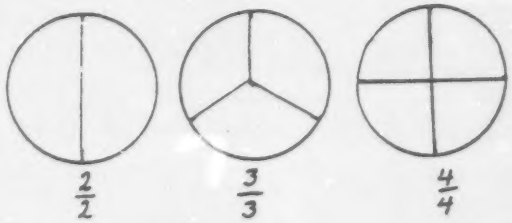
$\frac{12 \div 12}{36 \div 12} = \frac{1}{3}$ (see objectives 19 and 20)

Find all factors of the smaller number first and you can cease writing factors for the larger number when you begin to exceed the greatest factor of the smaller number.
Example: Find the GCF of 18 and 42.

Factors of 18 -- 2, 3, (6), 9, 18
Factors of 42 -- 2, 3, (6), 7, 14, 21

21 exceeds 18, so you can stop finding factors of 42. 6 is the GCF of 18 and 42.

$\frac{18}{42} \div \frac{6}{6} = \frac{3}{7}$ (see objectives 19 and 20)

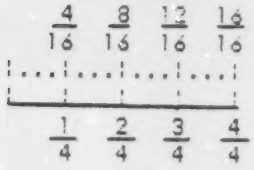
OBJECTIVE	PROCEDURE	ACTIVITY
18. Finding the LCM (Least Common Multiple) of two numbers by use of a matrix.	18. Place fractions to be renamed in matrix form. Begin first with numerator that has the smaller number and build your matrix. Then, when you work with the larger number, you can stop when you see you have the LCM. Now, you not only have your LCM but you have renamed your fractions and are ready to add or subtract, whatever your sign dictates.	18. Find the LCM of 1/4 and 1/3. $\frac{1}{4}$: $\frac{1}{4}$ $\frac{2}{8}$ $\frac{3}{12}$ = $\frac{3\times1}{3\times4}$ $\frac{1}{3}$: $\frac{1}{3}$ $\frac{2}{6}$ $\frac{3}{9}$ $\frac{4}{12}$ $\frac{5}{15}$ $\frac{6}{18}$ $\frac{7}{21}$ = $\frac{4\times1}{4\times3}$ Here 3 is smallest number in denominator so you find its equivalent first. $\frac{3}{12} + 4 = \frac{7}{12}$
19. Rename identity number of multiplication and division as a fraction.	19. a. Review identity number in multiplication and division of whole numbers to show that the product, or quotient, is the same as the number multiplied or divided by 1. Be sure to stress the answers as identical to the numbers that are being multiplied or divided by 1. b. Rename 1 as a fraction in many ways to show equivalence of fractions whose numerators and denominators are the same. Draw diagrams to show 1 renamed as a fraction.	19. a. Give facts involving identity number with whole numbers and have students give products or quotients: $\frac{8 \times 1}{9 \times 1} = \frac{8}{9}$ $\frac{8 \div 1}{9 \div 1} = \frac{8}{9}$ $\frac{2}{2}$, $\frac{3}{3}$, $\frac{4}{4}$ etc.  $\frac{2}{2}$ $\frac{3}{3}$ $\frac{4}{4}$

OBJECTIVE	PROCEDURE	ACTIVITY
20. Rename fractions, using the renamed identity number 1 as a fraction.	20. To rename a fraction you must multiply or divide the numerator and the denominator of your fraction by the same number.	20. a. Set up problems to illustrate division and multiplication by equivalence of 1.

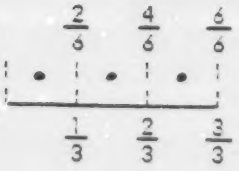
$$\frac{3}{4} \times \frac{4}{4} = \frac{12}{16} \qquad \frac{12}{16} \div \frac{4}{4} = \frac{3}{4}$$

$$\frac{2}{3} \times \frac{2}{2} = \frac{4}{6} \qquad \frac{4}{6} \div \frac{2}{2} = \frac{2}{3}$$

b. Draw diagrams to show that the renamed fractions are equivalent.



$\frac{3}{4} = \frac{12}{16}$

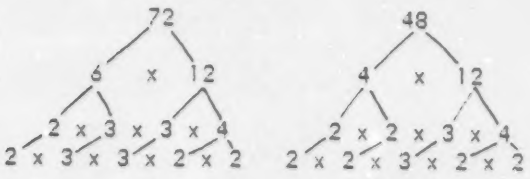


$\frac{2}{3} = \frac{4}{6}$

(objectives 19 and 20 are clarifications for objectives 14 through 18.)

21. Finding the LCM (Least Common Multiple) by prime factoring.	21. Prime factor the two denominators which you wish to rename for addition or subtraction operations. Present board work and interaction with students in explaining procedure. Example: Problem $\frac{16}{48} + \frac{12}{72}$ Find the LCM of 48 and 72 for the purpose of <u>adding</u> <u>16/48</u> and <u>12/72</u>.	

a.



b. Now you must match the prime factors so each denominator has the same set of prime factors.

$$\frac{16}{48} = \frac{16}{2 \times 2 \times 2 \times 2 \times 3} \;\Big|\; \begin{matrix} \times 3 \\ \times 3 \end{matrix} = \frac{48}{144}$$

$$+\ \frac{12}{72} = \frac{12}{2 \times 2 \times 2 \times 3 \times 3} \;\Big|\; \begin{matrix} \times 2 \\ \times 2 \end{matrix} = \frac{24}{144}$$

In comparing factors, 48 has one more two than 72. By placing a 2 with 72, both denominators have the same set of twos. But, 72 has one more 3 than 48. So, place a 3 with the primes of 48. Now both denominators have the same set of primes.

c. Note the vertical line drawn before the primes were added to the set of primes for each denominator. The line extends up into numerator area, too, because the prime factors added to the set of factors of a denominator must also be included as factors of the numerator of that fraction. Thus, we are multiplying the fraction by the equivalence of 1.

d. The LCM is either 3x48 or 2x72 because both products are the same. They have the same set of prime factors which gives them equal products.

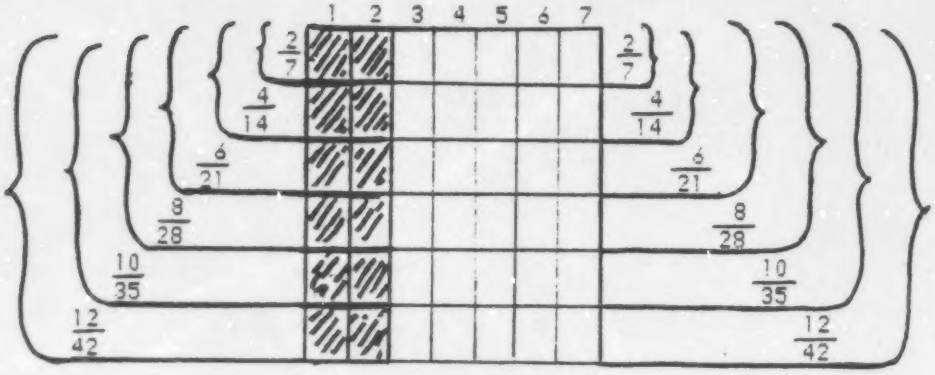
3x48=144 and 2x72=144

e. Rename the numerators and denominators and proceed with operation indicated by the sign.

$$\frac{48}{144}$$
$$+\ \frac{24}{144}$$
$$\frac{72}{144}$$

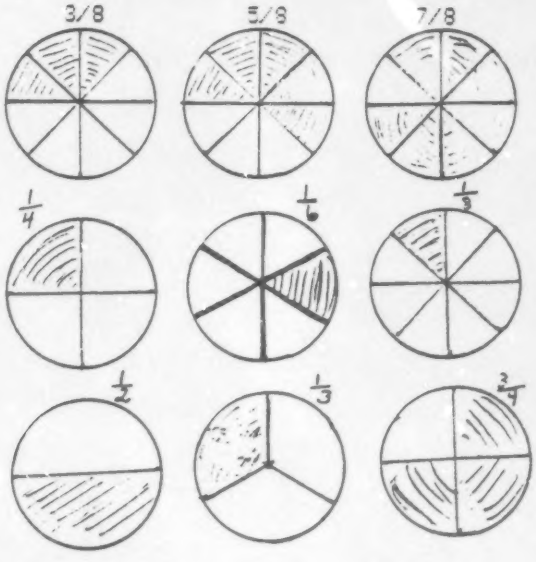
(See objectives 19 and 20 for clarification of renaming.)

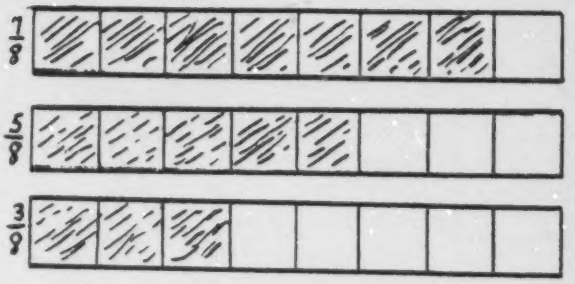
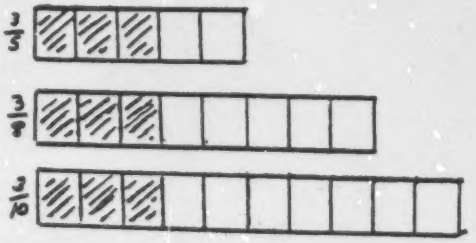
OBJECTIVE	PROCEDURE	ACTIVITY
22. Renaming is a prerequisite for addition and subtraction of fractions with unlike denominators.	22. Draw what you need in order to show equivalency of fractions once they are renamed. Be sure the child understands this concept before he/she is asked to work with pencil and paper. Drawing below shows one way to help the child understand equivalence after renaming is done.	22. Make a chart of equivalence after renaming is done. Have students draw charts showing equivalence of 2 fractions.



Equivalent fractions of 2/7 are:
$\frac{4}{14}, \frac{6}{21}, \frac{8}{28}, \frac{10}{35}, \frac{12}{42}$, etc.

23. Comparing and ordering of fractions.	23. a. Rename fractions so that denominators are the same. Then, compare numerators: Compare 7/8, 3/4, and 1/2. 7/8 = 7/8 3/4 = 6/8 1/2 = 4/8	23. Make visual representations to illustrate.)



OBJECTIVE	PROCEDURE	ACTIVITY
	1) When denominators are the same, the fraction with the larger numerator is the greater of the two fractions. Thus, we can order the fractions, once they are all renamed with a common denominator, by ordering the numerators. 7/8, 5/8, 3/8 (ordered from largest to smallest) -- all have same denominators, so ordering is done by ordering the numerators. 2) When the numerators are the same, the fraction with the smallest denominator is the larger of the two fractions. The smaller the denominator, when the numerators are equal, the larger the portion that the fraction represents. Example: 3/5, 3/8, 3/10. The numerators are the same. Therefore, one orders the denominators. The example is ordered from largest to smallest since the smallest denominator represents the largest portion when numerators are equal.	  Practice ordering fractions orally with explanations.
24. Adding fractions with like denominators.	24. When fractions have the same denominators, add numerators and place the sum over the denominator. Be sure you emphasize that the denominator names the fraction and we can only add fractions if they have the same name.	24. Much oral work is needed to lay the groundwork for renaming fractions with unlike denominators.

OBJECTIVE | PROCEDURE | ACTIVITY

Oral interaction such as:

(Student responds)

2 apples + 3 apples = ? (5 apples)

2 dogs + 4 dogs = ? (6 dogs)

Write on the board:

$\frac{2}{\text{(dogs)}} + \frac{3}{\text{(dogs)}} = \frac{5}{\text{(dogs)}}$

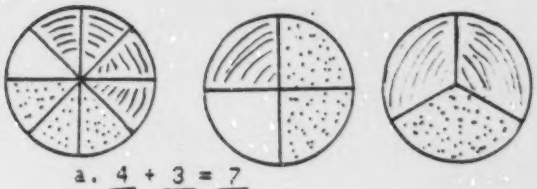
$\frac{2}{\text{(apples)}} + \frac{5}{\text{(apples)}} = \frac{7}{\text{(apples)}}$

$\frac{2}{(5)} + \frac{3}{(5)} = \frac{5}{(5)}$

$\frac{2}{(9)} + \frac{5}{(9)} = \frac{7}{(9)}$

Denominator is the name of the fraction.

a. b. c.



a. $\frac{4}{8} + \frac{3}{8} = \frac{7}{8}$

b. $\frac{1}{4} + \frac{2}{4} = \frac{3}{4}$

c. $\frac{2}{3} + \frac{1}{3} = \frac{3}{3}$

Illustrate adding of fractions. Use diagrams to show how you must change denominators so they are the same before you can add them.

$\frac{1}{2} + \frac{1}{4} =$

$\frac{2}{4} + \frac{1}{4} = \frac{3}{4}$

$\frac{1}{3} + \frac{1}{6} =$

$\frac{2}{6} + \frac{1}{6} = \frac{3}{6}$

$\frac{3}{6} = \frac{1}{2}$

OBJECTIVE	PROCEDURE	ACTIVITY
25. Subtracting fractions with like denominators.	25. a. Numerators are the numbers that one subtracts (smaller from larger), and places the difference over the denominator. b. Use the same technique as used in teaching addition of fractions with like denominators.	25. Similar diagrams may be used to show necessity for renaming fractions so one can subtract.
26. Adding fractions with unlike denominators.	26. a. Denominators must have same name before they can be added. Go through the whole procedure of renaming fractions. (Objectives 19 and 20, pgs. 71,72.) b. The LCM can be found by first seeing if one denominator is a multiple of the other. If it is, the multiple will become the LCM. Example: $\frac{2}{3} + \frac{5}{12}$ Twelve is a multiple of 3. Therefore 12 is the LCM. Rename 2/3 to 8/12 and proceed with the addition of the numerators: $\frac{8+5}{12} = \frac{13}{12} = 1\ 1/12$ c. The LCM can also be found by listing the multiples of both denominators of the fractions and circling the smallest one common to both. $\frac{x}{4}$ multiples of 4: (4, 8, (12), 16, 20, 24, 28) $\frac{x}{6}$ multiples of 6: (6, (12) 18, 24, 30) 12 is the LCM.	

OBJECTIVE PROCEDURE ACTIVITY

d. With a knowledge of primes, if one of the denominators is a prime number, the LCM is a product of the two denominators.

$\frac{x}{6}$ (6, 12, 18, 24, 30, 36, (42))

multiples of 6

$\frac{x}{7}$ (7, 14, 21, 28, 35, (42))

multiples of 7

7 is a prime, so the LCM is 7x6=42. By finding the set of multiples for each denominator, you can see that 42 is the LCM.

e. Once you have found the LCM, review the renaming of fractions. Use the knowledge that FxF=P and P÷F = F. We know the product is 42. So, product 42 divided by factor we know gives us the other factor.

Ex.:

$\frac{5 \times ?}{6 \times ?} = \frac{?}{42}$ (42÷6=7) $\frac{5x7=35}{6x7=42}$

$\frac{4 \times ?}{7 \times ?} = \frac{?}{42}$ (42÷7=6) $\frac{4x6=24}{7x6=42}$

Review Objectives 19 and 20.

f. Add numerators and place over the denominator. Since every fraction is a division problem, read from top to bottom and perform the operation

Answer to above problem e. is $\frac{59}{42}$. This is 59 divided by 42 which equals $1\frac{17}{42}$.

OBJECTIVE	PROCEDURE	ACTIVITY
27. Subtracting fractions with unlike denominators.	27. a. Denominators must have same name before they can be subtracted.	27. a. Practice finding the LCM of two numbers and renaming. Idea of Marsha Garner, found on p.71, could be used here for renaming.

b. Go through the whole procedure of renaming fractions (Obj. 19,20, pp.71, 72.)

c. See Obj. 26, p. 77, for finding LCM of two numbers. In this objective you would be finding the LCM of your denominators involved in the subtraction.

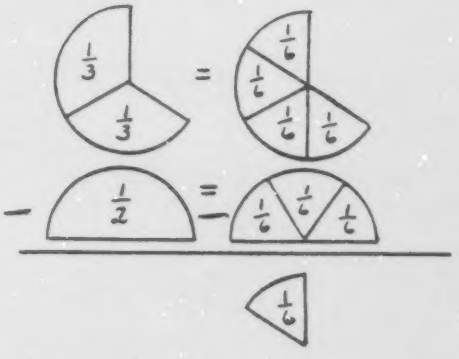
d. Once you find the LCM of the denominator, rename your fractions.
Ex.: 2/3 - 1/2
Since 3 and 2 are both prime numbers the LCM is the product of the 2 denominators. (3x2=6)

$$\frac{2}{3} = \frac{4}{6}$$
$$-\frac{1}{2} = \frac{3}{6}$$

e. Subtract the numerators and place answer over the denominator.

$$\frac{4 - 3}{6} = \frac{1}{6}$$

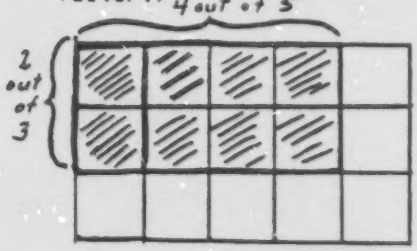
f. Draw circles to show that 2/3 and 1/2 are pictured differently. To subtract 1/2 from 2/3 you must find some way to divide both sections into smaller sections which are equal. When you find the LCM you have identified the size of those sections.



OBJECTIVE	PROCEDURE	ACTIVITY
28. Multiplication of fractions.	28. a. This is the easiest fraction operation taught by teachers (by rote): $\frac{\text{Numerator x numerator}}{\text{Denominator x denominator}}$ Ex. Use 2/3 and 4/5 as factors: $\frac{2 \times 4}{3 \times 5} = \frac{8}{15}$	

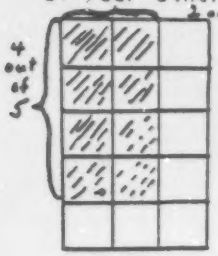
b. The child never fully understands what he/she has done. Let's try explaining first:

Let your first fraction be drawn vertically showing 2/3, or two out of 3 equal parts. Next, draw that same illustration five times (the number of times indicated by the denominator of the second fractional factor.)



c. Using the chart above, color in 2/3 the number of times indicated by the numerator of the second fraction. Specifically, you shaded in 2 out of 3, four out of the 5 times that you drew it. The shaded sections will give you the product of the two fractional factors.

shade: 2 out of 3
four out of 5 times
it is drawn and --
you get
8 out of 15 shaded in.

OBJECTIVE	PROCEDURE	ACTIVITY
	d. Show the commutative property of example in C above. 4/5 x 2/3 Draw a vertical illustration of 4/5. Repeat the illustration the number of times indicated by the denominator of your other factor.  Shade four out of 5.	
	e. Next, shade in the 4/5 the number of times indicated by the numerator of the other fractional factor, which is two. Specifically, you color 4/5 two out of the three times that you drew it. The shaded sections will give you the product of the two fractional factors. Shade 4 out of 5 two out of the three times. 4 out of 5 shaded in 2 out of 3 times = 8 out of 15 times	e. Draw various examples.
29. Cancellation -- a method used to simplify fractions prior to multiplying.	29. By a method of cross-cancellation one can simplify factors before multiplying, which will eliminate simplifying the product which is obtained. It may be necessary to perform several simplifications before multiplying.	29. Much board work is needed with teacher-to-pupil and pupil-to-teacher explanation of each step. Continually verify equivalence of the original fraction to its simplified form.

(Follow with use of commutative property to simplify, making the operation one of vertical cancellation only.)

Simplification may be vertical or diagonal. The important point is that a common factor must be found for a numerator and a denominator, whether they be in one factor (vertical) or two factors (diagonal).

Example of vertical cancellation:

$\frac{3}{4} \times \frac{8}{12}$ 4 is a common factor of both 8 and 12.

If we divide $\frac{8}{12}$ by $\frac{4}{4}$

it would look like this

$$\frac{2}{\frac{\cancel{8}}{\cancel{12}}}$$
$$3$$

Now our problem reads $\frac{3}{4} \times \frac{2}{3}$

We can no longer find a common factor vertically, but diagonally we can cross-cancel two ways, shown below:

$\frac{3}{4} \times \frac{2}{3}$ 2 is a common factor of 2 and 4.

When cross cancellation is done, it will look like this:

$$\frac{3}{\cancel{4}} \times \frac{\overset{1}{\cancel{2}}}{3} \quad \text{and the problem is now} \quad \frac{3}{2} \times \frac{1}{3}$$
$$2$$

Now we can see that diagonally, 3 is a factor of three so one more cross-cancellation will yield the following:

$$\frac{\overset{1}{\cancel{3}}}{2} \times \frac{1}{\cancel{3}} \quad \text{which gives}$$
$$1$$

$$\frac{1}{2} \times \frac{1}{1} = \frac{1}{2}$$

Now we have an answer in its simplest form.

OBJECTIVE	PROCEDURE	ACTIVITY
30. Use of commutative property to simplify factors before multiplying.	30. Since it does not change the product if I reverse my factors, I can switch either my numerators or my denominators so I can simplify my fractions vertically. Example: $\frac{3}{4} \times \frac{2}{3}$: a. By reversing my numerators I have $\frac{2}{4} \times \frac{3}{3}$ Both of the above problems, before cancellation is done, yield the same product $\frac{6}{12}$, which is not simplified. By vertically simplifying the fractional problem, after commutating numerators, I will have as the result: $\frac{\cancel{2}^{1}}{\cancel{4}_{2}} \times \frac{1}{\cancel{3}_{1}} = \frac{1}{2} \times \frac{1}{1} = \frac{1}{2}$ COMMUTATIVITY IS THE PROPERTY WHICH MAKES IT POSSIBLE TO CROSS CANCEL.	30. Work through many examples so the student fully understands why the commutative property works and how it works. Be sure the student understands that simplification of fractions is achieved by dividing by the equivalence of one, the identity number in division.
31. Reciprocal of a rational number -- and whole numbers written as fractions.	31. a. Reciprocal has the same derivation as reciprocate. Reciprocate means to give and receive mutually. "When I do something for you, you will in turn do something for me." A reciprocal in mathematics is a factor which will give a product of one when multiplied by the other factor.	31. Place numerous examples on the board and solicit examples from students.

OBJECTIVE	PROCEDURE	ACTIVITY
	A reciprocal of a fraction is the inverted form of that fraction. We reverse the numerator and denominator when we invert it. Ex.: $\frac{2}{3} \times \frac{3}{2} = 1$ $\frac{3}{2} \times \frac{2}{3} = 1$ 2/3 is the reciprocal of 3/2 and 3/2 is the reciprocal of 2/3	
	b. Whole numbers are written as fractions by placing them over 1. $9 = \frac{9}{1}$ $\frac{9}{1}$ is the reciprocal of $\frac{1}{9}$ and $\frac{1}{9}$ is the reciprocal of $\frac{9}{1}$ $\frac{1}{9} \times \frac{9}{1} = \frac{9}{9} = 1$	
32. Division of whole numbers by unit fraction. (Unit fractions have one as their numerator.)	32. a. Division of whole numbers by unit fractions is a multiplication operation. When we divide a number by 1/2, we will get twice as many section as we had before dividing, and the sections will be smaller than and part of the whole.	32. Draw examples to illustrate. a. $3 \div 1/2 = 6$. We get twice as many sections. $3 \times \frac{2}{1} = \frac{6}{1} = 6$
	b. To divide by 1/4 we will get 4 times as many sections as we had before, with each section being smaller than and part of the whole.	b. $3 \div 1/4 = 12$. We get 4 times as many sections. $3 \times \frac{4}{1} = \frac{12}{1} = 12$

OBJECTIVE	PROCEDURE	ACTIVITY
33. a. Dividing fractions with like denominators.	33. a. Division of fractions having like denominators may be solved by dividing the numerator of the fractional dividend, which is the first fraction in the division problem, by the numerator of the fraction which comes after the division sign and is known as the divisor. $\frac{8}{16} \div \frac{2}{16} = \frac{8 \div 2}{16 \div 16} = \frac{4}{1} = 4$ BE SURE a student sees that dividing denominators having the same name results in 1, which is the identity number in division and will not alter the quotient when the numerators have been divided.	33. a. Do board work and have children explain as they work. Example: $\frac{3}{8} \div \frac{6}{8} = \frac{3 \div 6}{8 \div 8} = \frac{\frac{3}{6}}{1} = \frac{3}{6} = \frac{1}{2}$ $\frac{15}{16} \div \frac{5}{16} = \frac{15 \div 5}{16 \div 16} = \frac{3}{1} = 3$
b. If denominators are not the same, a student could rename them and proceed as above.	b. Fractions could be renamed so the denominators are the same, and use the method described in (a). $\frac{2}{3} \div \frac{1}{5} =$ $\frac{10}{15} \div \frac{3}{15} = \frac{10 \div 3}{15 \div 15} =$ $\frac{3\ 1/3}{1} = 3\ 1/3$	
34. Divide fraction by whole number.	34. A. a. Make a model of a whole and divide it into the number of equal parts specified by the denominator of the dividend.	34.

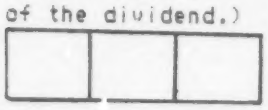
OBJECTIVE PROCEDURE ACTIVITY

Ex. 1:

$\frac{2}{3} \div 2 = \frac{2}{6} = \frac{1}{3}$

2/3 is the dividend and 2 is the divisor.

Make a model and divide it into thirds (denom. of the dividend.)

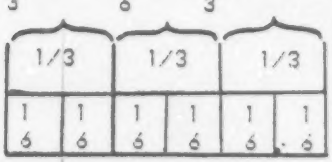


b. Divide each part into the number of equal sections indicated by the whole number, or divisor. (2)

Divide each third into 2 equal portions.

c. Next, divide 2/3 of the whole into the number of sections indicated by the divisor.

$\frac{2}{3} \div 2 = \frac{2}{6}$, or $\frac{1}{3}$ of the whole.



B. a. Make a model and divide it into the number of equal sections specified by the denominator of the dividend. (8)

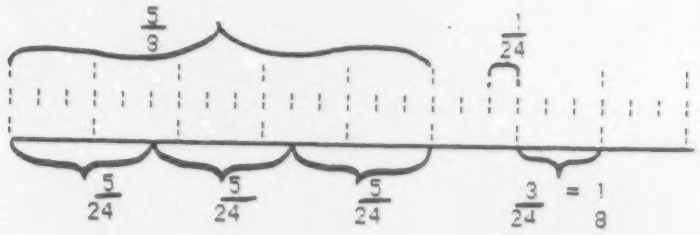
Ex. 2:

$\frac{5}{8} \cdot \frac{3}{1}$

b. Divide each equal part into 3 sections. Number specified by the whole number divisor.

OBJECTIVE	PROCEDURE	ACTIVITY

Now each section is 1/24 of the whole.



c. Divide 5 of those equal sections (number specified by the numerator of the dividend) into 3 equal parts, number specified by the divisor.

5 sections contain 15/24 of the whole.

When dividing them into 3 equal parts, each part will be 5/24 of the whole.

35. Rationale for "invert and multiply" in division of fractions, with unlike denominators.

35. a. In dividing by a fraction, the rationale for inverting the divisor and multiplying is the use of the identity number in division, which does not change the value of a number.

dividend divisor

$\frac{4}{6} \div \frac{1}{3}$ can be written as a fraction

$$\frac{\frac{4}{6}}{\frac{1}{3}}$$

since every division problem can be written as a fraction.

REMEMBER: You find an equivalent fraction by multiplying the numerator and the denominator by the same number.

35. Work out examples on the board and also have children do the same.

Study many examples to be sure the student understands the identity property.

OBJECTIVE	PROCEDURE	ACTIVITY

WITH THAT IN MIND:

b. After setting up the division problem as a fraction, multiply the numerator and the denominator by the reciprocal of the denominator. The reciprocal of $\frac{1}{3}$ is $\frac{3}{1}$.

Ex.:

$4/6 \div 1/3$

dividend $\frac{4}{6} \times \frac{3}{1} = \frac{12}{6} = \frac{12}{6}$ $\quad 12$

divisor $\frac{1}{3} \times \frac{3}{1} = \frac{3}{3} \quad 1 = 6 = 2$

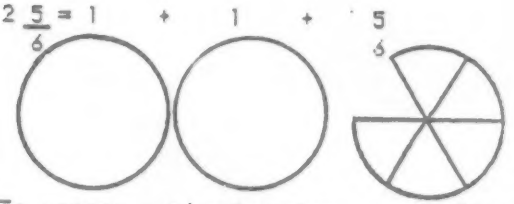
EMPHASIZE:
When both the numerator and the denominator are multiplied by the reciprocal of the divisor, it is simply multiplying them by one (the identity number in multiplication and division.)

c. Once a student understands why he inverts and multiplies when dividing by a fraction, he can simply invert and multiply without all the steps.

HOWEVER, understanding WHY is the key to all mathematics, and TIME spent in teaching WHY is well spent.

Concepts transfer into higher mathematics wich is more abstract. Rote learning does not transfer.

MIXED NUMBERS

OBJECTIVE	PROCEDURE	ACTIVITY
1. Mixed numbers.	1. A mixed number represents a whole number and a fraction. Explain a mixed number by writing and illustrating it in expanded notation. Ex.: $2\frac{5}{6}$ is a mixed number. $2\frac{5}{6} = 2 + \frac{5}{6}$ Use drawings to illustrate:  	1. Use fraction wheels, fraction charts, and number lines in presenting the lesson.
2. Renaming mixed numbers as improper fractions.	2. To rename a mixed number as an improper fraction, first rename the whole number as an improper fraction, using the same denominator as the denominator of the fractional part of the mixed number. Then add the renamed whole number to the existing fractional part of the mixed number. Using the mixed number $2\frac{5}{6}$, the denomination of fractional parts is sixths. Therefore, we rename 2 into sixths. Ex.: There are $\frac{6}{6}$ in 1. In 2 there are 2 times $\frac{6}{6}$ which means that $2 = \frac{12}{6}$	2. Place mixed numbers on the board. Choose students to dictate to you the procedure for renaming the mixed numbers into improper fractions as you write his/her dictates on the board. When a student renames a mixed number to an improper fraction, BE SURE that the student knows WHY he/she multiplies the denominator of the fraction times the whole number, adds to that the numerator from the fraction, and places the sum over the denominator of the fraction. (As you were taught.) DON'T teach renaming of mixed numbers the way you were taught until the student knows why he is taking the steps needed to rename his mixed number.

OBJECTIVE	PROCEDURE	ACTIVITY
	$(2 + \frac{5}{6}) =$ $\frac{12}{6} + \frac{5}{6} = \frac{17}{6}$ Use drawings to illustrate how whole numbers are divided into parts, using the same denominator as the fractional part of the mixed number.	The way you were taught to rename a mixed number should evolve from the teaching of the concept as shown in the procedure column.

$\frac{6}{6} + \frac{6}{6} + \frac{5}{6} = \frac{6 + 6 + 5}{6} = \frac{17}{6}$

$\frac{12}{6} + \frac{5}{6}$

ADDITION OF MIXED NUMBERS

3. Addition of mixed numbers with like denominators.	3. Before you perform the operation of addition, use expanded notation. whole number + fraction + whole number + fraction whole number + fraction (rename)	3. Do extensive board work. Verbalization of steps by the student is important, as teacher places work on board.

a. Using $2\frac{2}{3} + 1\frac{2}{3}$, expand to:

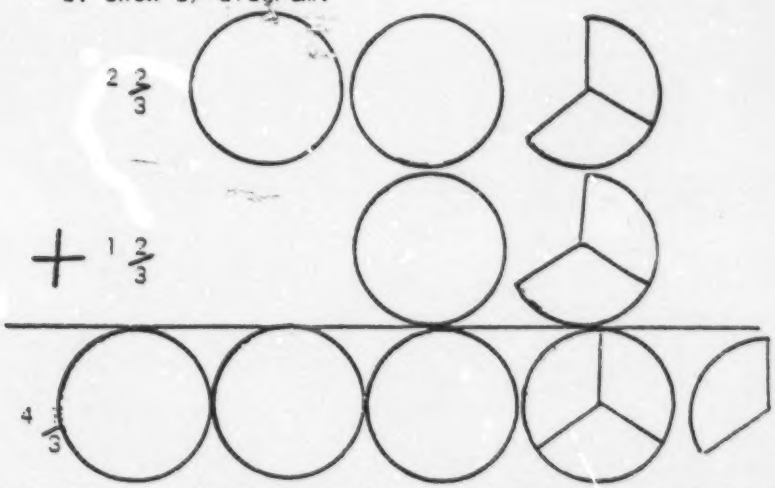
$2 + \frac{2}{3}$ Add fractions and whole numbers, performing the operation from right to left.

$+ 1 + \frac{2}{3}$

$3 + \frac{4}{3}$

When fractional part is equal to one, or more than one, rename the fraction.

b. Rename the fraction $\frac{4}{3}$ to $1\frac{1}{3}$

OBJECTIVE	PROCEDURE	ACTIVITY
	c. Rename the entire sum, using the associative property: $3 + (1 + \frac{1}{3}) =$ $(3 + 1) + \frac{1}{3} = 4 + \frac{1}{3} = 4\frac{1}{3}$	
	d. Show by diagram: 	
4. Addition of mixed numbers when the fractions have different denominators.	4. If the fractional parts of mixed numbers have different denominators, the fractions must be renamed to have like denominators and the procedure will then be the same as addition of mixed numbers with like denominators, Objective 3. Turn to fraction section, pages 71-74 and review renaming of fractions.	4. Be sure student knows the use of the parentheses in grouping of numbers.
SUBTRACTION OF MIXED NUMBERS		
5. Subtraction of mixed numbers without renaming of fractions or whole number.	5. See subtraction of whole numbers, pg.17 and subtraction of fractions without renaming, pg.77. The same procedure applies. Subtract fractions, then subtract whole numbers.	5. Student always works from right to left in subtraction.

OBJECTIVE	PROCEDURE	ACTIVITY
6. Subtraction of mixed numbers when fractions must be renamed.	6. If fractional parts of mixed numbers have different denominators, students must first rename their fractions so they have like denominators. Proceed with expanded notation as in addition, and perform the subtraction from right to left. Example: $2\frac{3}{4}$ $-\ 1\frac{1}{2}$ expanded: $2 + \frac{3}{4}$ $-\ 1 - \frac{1}{2}$ renamed fractions: $2 + \frac{3}{4}$ $-\ 1 - \frac{2}{4}$ $1 + \frac{1}{4} = 1\frac{1}{4}$	
7. Subtraction of mixed numbers when it is necessary to rename the whole number.	7. Begin instruction by naming the parts of a subtraction problem as Sum - Addend = Addend. If the fractional part of a mixed number cannot be subtracted from the fractional part of another mixed number, it will be necessary to "borrow" one from the whole number of the Sum and rename that one as a fraction, giving it the same name as the fractional part of the Sum.	

OBJECTIVE PROCEDURE ACTIVITY

Always perform the subtraction from right to left.
Ex.:

$$\begin{array}{rl} \text{Sum} & 3\tfrac{1}{3} \\ -\ \text{Addend} & -\ 1\tfrac{2}{3} \\ \hline \text{Addend} & ? \end{array}$$

Follow this example in explaining the subtraction procedure, step by step:

You cannot subtract 2/3 from 1/3. Therefore, it is necessary to "borrow" one from the 3 in the Sum and rename it into thirds, which is the name of the fractional part of the Sum.

Step a: (State the problem)

$$\begin{array}{rl} \text{Sum} & 3\tfrac{1}{3} \\ -\ \text{Addend} & -\ 1\tfrac{2}{3} \end{array}$$

Step b: (Borrow 1 from the 3) and use associative property to group 1 with the fraction.

$$\text{Sum} \quad 2 + (1 + \tfrac{1}{3})$$

Step c: (Rename the 1 into thirds.)

$$\text{Sum} \quad 2 + (\tfrac{3}{3} + \tfrac{1}{3})$$

Step d: (Add $\frac{3}{3}$ and $\frac{1}{3}$ which equals $\frac{4}{3}$)

OBJECTIVE	PROCEDURE	ACTIVITY

Step e: (Set up complete problem again, after "borrowing" and renaming.)

$$\begin{array}{rl} \text{Sum} & 2 + \frac{4}{3} \\ - \text{ Addend} & - 1 - \frac{2}{3} \\ \hline \text{Addend} & 1 + \frac{2}{3} \end{array}$$ which is written as 1 2/3.

The difference found when subtracting is the missing addend of an addition problem.

Step f: Eventually, renaming will be done without using expanded notation, and the process will be shortened.

MULTIPLICATION OF MIXED NUMBERS

8. Multiplication of a mixed number times a whole number.

8. Rename the mixed number, making it an improper fraction. (See objective 2 of Mixed Numbers, pg. 89).

Following renaming of the mixed number as an improper fraction, place the whole number over 1, making it also an improper fraction, then multiply.

Follow procedure given on page 80, Objectives 28-30, Multiplication of Fractions.

Ex.: $2\frac{1}{8} \times 4$

a. Rename $2\frac{1}{8}$

Procedure: There are $\frac{8}{8}$ in 1:

In 2 there are $2 \times \frac{8}{8}$

$2 \times \frac{8}{8} = \frac{16}{8}$

$\frac{16}{8} + \frac{1}{8} = \frac{17}{8}$

8. Board work with teacher - pupil interaction.

OBJECTIVE	PROCEDURE	ACTIVITY

b. Rename 4 as an improper fraction by placing 4 over 1. $\frac{4}{1}$

c. Multiply $\frac{17}{8} \times \frac{4}{1}$

Follow Objectives 28-30, pages 80 - 83, for multiplication of fractions.

9. Multiplication of a mixed number times a mixed number.

9. Before you multiply two mixed numbers, rename both of the factors, making them improper fractions.

After the mixed numbers have been renamed, proceed with the multiplication operation as explained in Objectives 28-30 on pages 80-83, for Multiplication of Fractions.

Ex.: $2\frac{1}{4} \times 3\frac{1}{3}$

a. Rename both mixed numbers as improper fractions:

$\frac{9}{4} \times \frac{10}{3}$

b. By cancellation:

$$\frac{\cancel{9}^{3}}{\cancel{4}_{2}} \times \frac{\cancel{10}^{5}}{\cancel{3}_{1}} = \frac{15}{2} = 7\frac{1}{2}$$

(See Fractions, Objective 29, pg. 81).

c. Use of commutative property, see switching denominators:

$$\frac{3}{9} \times \frac{10}{3} = \frac{15}{4} = 7\frac{1}{2}$$

$$\frac{1}{2}$$

(See Fractions, Objective 30, pg. 83).

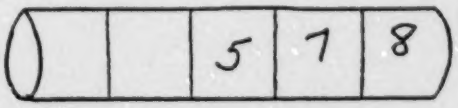
DIVISION OF MIXED NUMBERS

OBJECTIVE	PROCEDURE	ACTIVITY
10. Division of mixed number by a whole number.	10. Explain division as Product ÷ Factor = Factor. First step is to rename the mixed number, making it an improper fraction. (See Objective 2, pg. 89). Next, rename the whole number, making it an improper fraction by placing it over 1. Follow procedure for multiplying the product by the reciprocal of the factor used as the divisor, which is the factor immediately following the division sign. Follow multiplication of fraction directions (Objective 28, Fractions, pg. 80.) Ex.: Product ÷ Factor = Factor Mixed Whole $2\frac{2}{3} \div 3$ Rename both product and factor used as the divisor: Product ÷ Factor = Factor $\frac{8}{3} \div \frac{3}{1}$ Multiply product by reciprocal of factor used as the divisor. $\frac{8}{3} \times \frac{1}{3} = \frac{8}{9}$ The product in the division problem became a factor and the missing factor is now a product, or the answer to a division problem in fractions.	10. Review reciprocal. Review division of fractions.

OBJECTIVE	PROCEDURE	ACTIVITY
11. Division of whole number by a mixed number.	11. Follow steps above for renaming of mixed number and whole number, making them both improper fractions. Follow procedure for multiplying the product by the reciprocal of the factor immediately following the division sign. renamed (P ÷ F)=(P ÷ F)=(P x reciprocal of the F which is the divisor) Ex.: $(4 \div 2\frac{2}{3}) = (\frac{4}{1} \div \frac{8}{3}) = (\frac{4}{1} \times \frac{3}{8})$	
12. Division of mixed number by a mixed number.	12. Rename the Product and the Factor used as a divisor, making both of them improper fractions. Proceed as you would with division in Objectives 10 and 11.	

OPERATIONS USING DECIMALS

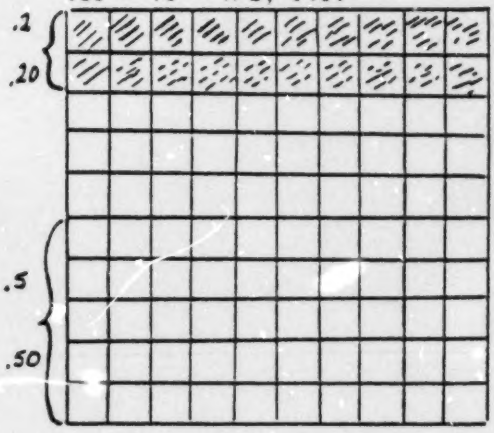
ADDITION

OBJECTIVE	PROCEDURE	ACTIVITY
1. Addition of decimal fractions.	1. In addition of decimals, numbers should be lined up so that the decimal points of each number appear one under the other. By lining up the decimal points, you are lining up place values, adding tenths to tenths, hundredths to hundredths, thousandths to thousandths, etc. Since place values of digits in a decimal fraction are arrived at by the same procedure as in the place value of digits in a whole number, the addition operation will not change. Use many examples to show how .010 becomes .01, how .10 becomes .1 etc. Draw a grid and show how renaming is done in the same manner as it is with addition of whole numbers. Example problem:	1. a. Do board work with grids. Have students explain why they place numbers in certain place values on the grid. b. Again refer to gasoline pumps, both for gallons and price of gasoline, and note how the place value numerals change. c. The odometer may be again referred to, or odometers may be made by placing number strips on cardboard cylinders.  Each place value strip contains 0 through 9. Once a place value strip runs through 9, a zero appears in that place value strip and the place value strip on its immediate left adds on one. In above odometer, when the 8 in the ones column becomes a 9 and then a 0, the 7 in the tens place will become an 8, etc.

Tens	Ones	Tenths	Hundredths	Thousandths
	2.	9	4	2
	5.	4	9	8
	7.	3	6	7
	8.	8	5	6
2	4.	6	6	3

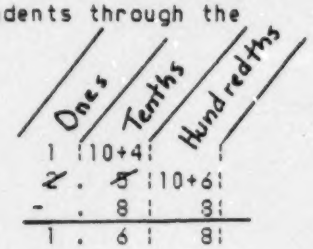
OBJECTIVE	PROCEDURE	ACTIVITY
	Walk pupils through each step of the problem as is done below for the sample problem above. When adding numbers in the thousandths place, the sum is 23 thousandths. .023 is broken down in expanded notation to be 0 tenths, 2 hundredths and 3 thousandths. You place the 3 in the thousandths place and "carry" the 2 into the hundredths place. When adding the numbers in the hundredths place, the sum is 26 hundredths. .26 is noted as 6 in the hundredths place and "carry" 2 into the tenths place. When adding the numbers in the tenths column, you get a sum of 26 tenths, which is 2 and 6 tenths. Place the 2 in the ones column on the left side of the decimal and the 6 in the tenths place on the right side of the decimal. When adding the ones column, you get a sum of 24 ones. In expanded notation this is 2 tens and 4 ones. So, you place 4 in the ones column and "carry" the 2 tens to the tens column. The decimal point in the answer is still directly under the decimal point in the addends. Renaming, or "carrying," does not change the position of the decimal point. When you change a decimal point, all place values change.	Let students explain problems to you, walking through all the steps as they go. References: Use "Math No Mystery", Lessons 19, 20, SOITA, EMC. "Mathways", Lesson 1, SOITA, EMC.

SUBTRACTION

OBJECTIVE	PROCEDURE	ACTIVITY
2. Zero as a place holder.	2. Zeros may be used at the end of a decimal fraction without increasing the value of a number. Therefore, in the example below, you may place zeros in all place values not holding a number. Example: 3 . 2 1 8 4 . 5 3 1 . 4 Using example above, insert zero place holders and add: 2 . 6 0 5 3 . 2 1 8 4 . 5 3 0 1 . 4 0 0 11 . 7 5 3	2. a. Have students prove equivalency of numbers such as .50, .500, .5, by dividing by equivalence of one. $\frac{500}{1000} = \frac{50}{100} = \frac{5}{10}$ b. Draw 10x10 grids divided into hundreds and show equivalency of tenths to hundredths: .20 = .2 .50 = .5 = 1/2, etc. 
3. Subtraction of decimal fractions.	3. In subtraction of decimals, numbers should be lined up so that the decimal points of each number appear one under the other. By lining up the decimal points, you are lining up place values, subtracting tenths from tenths, hundredths from hundredths, thousandths from thousandths, etc. Since the decimal fraction is an extension of the base 10 system, the subtraction operation will proceed the same for tenths, hundredths, and	3. a. Review place value of decimal system on page , Objective 11, Numbers and Numeration. b. Do board work. Place a grid on the board. Walk students through every step. Have students also walk through each step as they do board work.

thousandths as it did for tens, hundreds, and thousands.

Ex.: Make a grid on the board. Set up problems such as the one below and walk students through the steps.



Explanation of steps for above example:

a. You cannot subtract 8/hundredths from 6 hundredths. "Borrow" one/tenth from the 5/tenths. One/tenth becomes 10/hundredths.
$\frac{10}{100} + \frac{5}{100} = \frac{16}{100}$.
Now you can subtract 8/hundredths from 16/hundredths which leaves 8/hundredths to place in the hundredths place of your difference.

b. Now you have 4 left in the tenth's place. You cannot subtract 8/tenths from 4/tenths, so you "borrow" one from the ones column. One = 10/tenths.
$\frac{10}{10} + \frac{4}{10}$ = 14/tenths,
from which you can subtract 8/tenths. $\frac{14}{10} - \frac{8}{10} = \frac{6}{10}$
Place 6 in the tenths column of the difference.

c. One is left in the ones column, so you bring it down in the difference, since you have nothing to subtract from it.

MULTIPLICATION

OBJECTIVE	PROCEDURE	ACTIVITY
4. Multiplication of decimal fractions. (Review the multiplication of common fractions, Objective 27, pg.80.)	4. In multiplying a decimal times a decimal, relate the decimal fractions to the common fractions so the student will understand how he/she arrives at the number of decimal places in the product. In explanation: tenths x tenths = hundredths $\frac{1}{10} \times \frac{1}{10} = \frac{1}{100}$ .1 x .1 = .01 This is why a one-place decimal times a one-place decimal gives a two-place decimal as a product. tenths x hundredths = thousandths $\frac{1}{10} \times \frac{1}{100} = \frac{1}{1000}$ .1 x .01 = .001 This is why a one-place decimal times a two-place decimal gives a three-place decimal as a product. DON'T TEACH THE CHILD TO COUNT THE NUMBER OF PLACES TO THE RIGHT OF THE DECIMAL POINT IN THE TWO FACTORS TO OBTAIN THE NUMBER OF DECIMAL PLACES IN HIS PRODUCT. TEACH HIM HOW A PRODUCT IS NAMED. Have student verbalize the reason for the number of places in his decimal fraction.	4. a. Board work - oral and visual practice. b. Have students use same procedure for explaning the number of places in a decimal fraction. c. Cards with 10ths, 100ths, 1000ths, etc., may be used by students to show the teacher the name of the product for a problem written on the board. d. Cards may also be made with 1, 2, 3, 4, 5, on them. Student holds a card up telling how many decimal places there are in the product of a problem either dictated to them or written on the board. Ex.: If I multiply a three place decimal by a two-place decimal, how many digits will be to the right of the decimal point in the product. (The student will hold up a 5) NOTE: Ignore the decimal points until you obtain the product. Do not place decimal points in partial products.

OBJECTIVE	PROCEDURE	ACTIVITY
	N.B., RELATE PLACES TO THE RIGHT OF THE DECIMAL POINT TO ZEROS IN THE DENOMINATOR OF THE COMMON FRACTION. $\frac{1}{10}$ is a one-place decimal fraction (.1) and has one zero in the denominator of the common fraction. $\frac{1}{100}$ is a two-place decimal fraction (.01) and has two zeros in the denominator of the common fraction, etc.	
5. Prefixing zeros when product does not contain enough decimal places.	5. Example: .23 x .36 138 69 828 (incomplete) Once the multiplication is finished we must decide where the decimal point will be. Reasoning: hundredths x hundredths = ten thousandths There are 4 places in ten thousandths. There are only 3 places in the product of the example. Where do you add a zero and still be able to read the number 828 ten thousandths? If you annex a zero (8280), it will read 8,280 so, when you need to add numbers to satisfy a requirement for decimal places prefix them before the highest place value but after the decimal point (.0828). Now your number will read 828 ten thousandths.	5. Review Numbers and Numeration, Obj. 18, pg. 11. Work many problems and let students deduce the rule "counting places to left of decimal in both factors equals number of places to right of decimal in the product."

OBJECTIVE	PROCEDURE	ACTIVITY
6. Estimating where a decimal point belongs in the product when there are whole numbers and decimal fractions involved in the factors.	6. When multiplying mixed numbers involving decimal fractions, cover the place values to the right of the decimal points in the factors. This exposes only the whole numbers in the factors, which you round off to the nearest multiples of 10 and multiply. This will yield an approximation in the whole number of the product by giving the number of digits in the whole number, and a product with a value in the neighborhood of the whole number. Example: 783.56 round off to 800 x 23.22 round off to x20 16,000 An estimate will tell you that the product of the number above will be a 5-digit whole number with an approximate value of 16,000. The actual product is 18,194.2632, which includes a 5-digit whole number, close to 16,000.	6. Spend some time doing work on the board. Give the students problems. Let them round off, estimate with the idea of approximation, then check out the validity of his/her answers by use of calculators. If students have calculators, let them use them to see if they arrive at a ball-park product. Or, choose students to use your calculator to check answers. We are not seeking the correct calculation here, done by the student, but the ability to arrive at a valid answer. Don't ruin a valuable lesson by expecting a child to perform such calculations with pencil and paper.

DIVISION

OBJECTIVE	PROCEDURE	ACTIVITY
7. Division of decimal by a whole number.	7. When dividing a decimal by a whole number, place the decimal point in the quotient directly above the decimal point in the dividend. Do this before you proceed with the division operation, then proceed as if there were no decimal points involved.	7. Remind students that decimal points do not interfere with division process since any place value in decimals is ten times the value of any place on its right.

OBJECTIVE	PROCEDURE	ACTIVITY
8. Division of a decimal or whole number by a decimal.	8. When dividing by a decimal it is necessary to remove the decimal point from the divisor. If a one-place decimal is the divisor, you will remove the decimal point by multiplying your divisor by 10. This will be shown by moving the decimal point one place to the right. (See pg. 7, Objective 12 in this unit.) Thus we rename the divisor. When we rename the divisor, we must also rename the dividend by multiplying it by 10. Here, you must reteach that a division problem is a fraction. The numerator of a fraction is the dividend and the denominator is the divisor. In renaming a common fraction you must multiply or divide the numerator and the denominator by the same number. Since moving a decimal point to the right in a number is a multiplication process; and,	8. Relate renaming of decimals in division to renaming of fractions.

OBJECTIVE	PROCEDURE	ACTIVITY
	Since we are renaming the divisor, we must also rename the dividend. We multiply the dividend by the same number we used to multiply our divisor. Move the decimal point the same number of places to the right in the dividend as was required to remove the decimal point from the divisor. Thus, both the dividend and the divisor have been multiplied by the same multiple of 10 and have not changed the value of the fraction. (See Fractions, Objectives 19 and 20, pg. 71,72.)	
9. Estimating the number of digits in the whole number of a quotient when a dividend includes a decimal and rounding off the divisor and dividend to the nearest multiple of 10, then dividing to obtain an approximate quotient.	9. Using an example such as $.76\overline{)235.436}$: Step 1: Remove the decimal points from the dividend and divisor by multiplying each by 100. This is done by moving the decimal points in both dividend and divisor two places to the right. (Explained in Objective 12 of this unit, pg. 108). Now, your problem will read $76\overline{)23543.6}$ Step 2: Round 76 to 80 and 23543 to 24000 and divide. You will get a quotient of 300 because 300 x 80 = 24000. This will tell you that your quotient will be a 3-digit number and will be approximately 300.	9. Place problems on the board for students to round off the dividends and divisors and estimate quotients. If they have calculators, let them feed the problem into the calculator to see if they have estimated their answer somewhere near the actual quotient. REMEMBER: The skill we are seeking here is one of rounding off and estimating - not calculating. SO - The mastery of these objectives becomes a game -- not a chore.

OBJECTIVE	PROCEDURE	ACTIVITY
	Step 3: With calculators perform the division problem of 23543 76. You will receive a quotient of 309.7 R64. 309 is a 3-digit number and is close to 300.	
10. Annexing zeros to dividend when there are not enough places to move decimal point over.	10. When dividing by a decimal that has more place values than exist in the decimal places of the dividend, it will be necessary to annex zeros. Example: $.76\overline{)25.1}$ The divisor is a two-place decimal. The dividend is a one-place decimal. To remove the decimal point from the divisor you multiply it by 100. You must also multiply the dividend by 100. 25.1 x 100 = 2510 You must annex a zero and place the decimal point to the right of the zero. $.76\overline{)2510.}$ Once you find where the decimal belongs in the dividend, immediately place your decimal point in the quotient directly above the newly located decimal in the dividend.	
11. Rounding of numbers.	11. You must always have a place value to the right of the place value you are rounding to.	
a. to nearest whole number.	a. When rounding to the nearest whole number, circle your whole number and the place on its right (tenths place). Ex. 27.6 If the number to the right (tenths place) is 5 or	

OBJECTIVE	PROCEDURE	ACTIVITY
		greater, increase by one the number to the left of it and drop the number in the tenths place (28).
b. to nearest tenth.	b. When rounding to the nearest tenths, circle the tenths and hundredths places. Ex. 23.(63) If the number to the right in your circle is 5 or less leave the number on the left as it is and drop the number in the hundredths place (23.6).	
c. to the nearest hundredth.	c. When rounding to the nearest hundredths, circle the number in the hundredths place and the place of value to its right (thousandths place). Example 1: 2.7(68) The thousandths place is greater than 5 so you increase the hundredths place by one and drop the number in the thousandths place (2.77). Example 2: 2.7(63) The thousandths place is less than 5 so you leave the hundredths place as is and drop the number in the thousandths place (2.76).	
12. Carrying answers in division to nearest	12.	
a. tenths	a. Giving answers to nearest tenths requires finding a quotient through the hundredths place (2 places).	

OBJECTIVE	PROCEDURE	ACTIVITY
b. hundredths	b. Giving answers to nearest hundredths requires finding a quotient through the thousandths place (3 places).	
c. thousandths	c. Giving answers to nearest thousandths requires finding a quotient through the ten-thousandths place (4 places.)	

SUGGESTIONS FOR TEACHING THE BASIC FACTS OF ARITHMETIC

Child must understand material before beginning to memorize facts. It is important for the child to have a clear idea of the nature of his/her goal and the reason it is appropriate to memorize facts (time saving). A pupil must be able to validate a fact through manipulatives or pictorial presentation.

Drill instructions:

a. Use visual and auditory drill with intent to memorize.
b. Keep drill sessions short.
c. Have some drill almost every day.
d. Memorize only a few facts as a given lesson.
e. Constantly review previously memorized facts.
f. Control the number of facts added so success with old facts will help keep up interest and confidence.
g. Present memorization of facts as a challenging, fun, and satisfying activity.
h. Actively use commutative relationship and application of 0 and 1 properties to help pupils attain mastery.
i. Provide variation which has immediate feedback.
j. Monitor effort and progress of each child closely.
k. Praise good effort.
l. Keep record of progress for each child.

(from Developmental Computational Skills, 1978 NCTM Yearbook, Reston, Va., pgs. 52-58.)

Each child should have his/her own set of flash cards.

Each child should be paired with another child to practice facts.

Have child prepare a box in which to keep flash cards he/she does not know. When a fact is mastered, remove it from the container and give the child verbal recognition. Ultimate goal is to remove all unknown facts from the container before the year's end.

REMEMBER: TOO OFTEN it is TOO SOON to drill.

but

DRILL has not been OUTMODED,

and

must be done OVER and OVER - - -

FASTER and BETTER!

FACT PRACTICE

LEARNING BY GROUPING

Before memorization, supervise use of manipulatives so children understand that when they work with facts they are working with groups.

Give students a paper that is marked into two sections when they begin addition and subtraction facts. Use sets of manipulatives, preferably 20 popsicle sticks, or something which can be grouped in bundles of ten if need be.

If pupil is adding, have him/her place one group representing one addend on one side of the sectioned paper and on the other section place a group representing the other addend.

As pupil moves a group of 6 together with a group of 4 the meaning of the fact he/she is asked to memorize can readily be seen.

Likewise, if asked to show 10 6, the pupil will form a group of 10 on one side of the sectioned paper and will see that four is the group left when a group of 6 is moved to the other section of the divided paper.

Six objects are left in a group that originally had 10 objects in it before 4 were placed in a separate group.

Have the child recite all four facts that he/she needs when these manipulatives are performed.

FIGHTER PILOT (any operation)

All students are pilots. A designated pilot mans the fighter plane. He/she stands. The teacher shows a flash card. If the "seated" pilot answers correctly before the fighter pilot, the seated pilot becomes the fighter pilot, stands, and challenges the next seated pilot. Someone keeps score to see which pilot "shoots down" the most planes. The pilot with the highest 2score becomes the ACE PILOT and wears a pair of MATH WINGS until the next contest. The winner's name is also entered on a displayed list of ACE PILOTS.

To be sure that slower students have a chance to become ACE PILOTS, cooperate with another class and do some ability grouping of students to compete against. This way you can have those who are slower compete against a more homogeneous group and all can achieve a modicum of success.

SIT DOWN RELAY (can be done with facts or problems of any difficulty)

Be sure the class has been divided into even-numbered rows of students of comparable abilities, heterogeneously grouped in each row. Give each row a paper with the same set of problems on it (number of problems depends upon number of students in each row). Or, you could designate problems on a page in a text. The first person in the row works the first problem; the second, the second problem, etc. The first row finished, with all problems correct, gets one point. The second row finishing with all correct gets two points, etc. The procedure may be repeated that day, or another day. A running score is kept, with the teams remaining the same. At the end of a grading period the team with the lowest score gets some special reward.

Have students bring in old sets of playing cards. Use all except the face cards.

SPEEDY SOLUTION (may also be adapted for multiplication)

Use two sets of cards and 3 players, one being the referee and score keeper.

Shuffle the cards, a deck for each player.

Both players turn over a card. The first one to give a sum or difference of the two cards turned over, (depending upon what operation is being used) gets a point.

Shuffle cards again, after all have been used at least once. Set game rule for winning before game starts.

TEAM WORK

Divide the class into teams of even abilities. Make each team responsible for all of its members. Team learning all their facts first (in whatever operation is being studied), gets some special activity.

Later, make new teams and give the students having difficulties to the team that is stronger, or has those members on it who have to win. They will drive the slower students to learn their facts.

TUNE TO "THE BUNNY HOP" (have a large number line on the floor)

1	2	3
Start with number (4)/(6)	(6)/(4) was your second addend.	Subtraction undoes addition.
It is addend number one.	Let's backward hop that (6)/(4).	They're inverses so we say.
Hop to the right (6)/(4) spaces.	You're now at addend (4)/(6)	If you understand the the relation.
Then tell us the sum. (10)	And subtraction did the trick.	Our family game we'll play.

First number in parenthesis is used for first singing of verses.
Second number in parenthesis is used for second singing of verses.

Sing the song, using any two numbers. Have the child give the four complete facts he can discover if he reverses his numbers in the song.

Example: 4 + 6 = 10 10 − 6 = 4
6 + 4 = 10 10 − 4 = 6

WHAT'S MY N?

Choose sides, as homogeneously grouped as possible. Leader (either pupil or teacher) says.

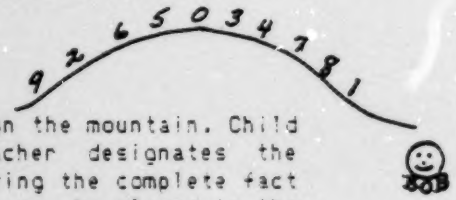
"If n + 12 = 17, what's my n?"
"If 6 − n = 4, what's my n?"

Remember who your lead student was. If a child misses, he goes to the end of the line. Count the number of students who had to go to the end of the line (count those twice who went to the end of the line twice, or as many times as they had to go to the end of the line). Team moving the least number of students to the end of the line wins.

GEOBOARD DISCOVERY

By placing rubber bands around various groupings of pegs on the geoboard, a child can discover various facts. If he bands three groups of fours, he will discover a sum of 12 (4+4+4) or the product of 12, (3x4).

OVER THE MOUNTAIN



Arrange all 10 digits on a mountain. Call out a number on the mountain. Child will add that number (or multiply, or whatever teacher designates the operation to be) to all the numbers on the mountain, saying the complete fact (9+9=18). A smiley face, with the child's name on it, may be placed in the valley beside the mountain, if he gives all correct facts.

POPOVER (addition and subtraction)

Two students and a person who has instant recall of the facts being used can play. Use all the number cards 2 through 10. Decide the rules governing your game, before a card is turned over. A player picks up a card and gives a problem which would yield the number the player picked up. After using all the cards, you can start over, but the same problem cannot be given for the answer cards.

Some examples a person might use are:
My card is 5 more than 4. Other player names the card as 9.
My card is 2 less than 6. Other player names the card as 4.

After the game is understood, you do not have to say "My card is". Just say "4 greater than 2", "3 times 3", etc. Score one for each correct answer.

SIDESWITCH

Divide class into two sides and line them up in pairs, as homogeneously paired as possible. Show a flash card. The first one of the pair at the head of the line who gets the answer takes the other person in the pair over on his/her side and they go to the end of the line. Team with most persons switched over wins.

SPUD MASH

Two teams form separate circles. Each team has a potato which it starts passing in its circle. When the teacher calls "MASH", the children with the potatoes must give the answer to a math fact that the teacher calls out. First team player to answer gets a point for his/her team. If, however, the answer is wrong, the team must deduct a point from its score.

Time limit or winning score should be set before the game begins.

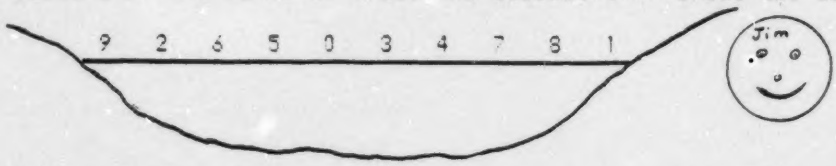
MUSH POT TAG (for any operation)

Two or three children volunteer to be in the "mush pot", which is inside the circle. Another pupil is chosen as the challenger. The pupil skips around the circle and suddenly stops behind a player to challenge him/her with a problem. The player who is challenged must give the correct answer before someone in the mush pot does. If successful, he/she becomes the challenger and skips around the circle to challenge another player with another problem. If challenged pupil fails to give the correct answer before the person in the "mush pot", he/she must exchange places with the pupil in the mush pot.

Example: What is 4x5? What is 9-3? What is 27÷3? What is 6+5? etc."What is" can be dropped once game is understood.

BRIDGING THE CANYON (adaptable to multiplication)

Arrange all 10 numerals on a bridge that supposedly spans a canyon. Teacher calls out a number to be added to those on the bridge. A child who fails to give correct answers could fall in the canyon. Or, to keep it positive, smiley faces could bear the names of those who successfully cross the canyon.



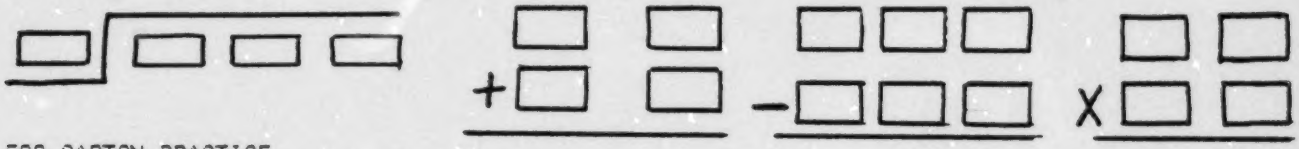
CLAP-SNAP-CLAP-CLAP-CLAP=4 (or something else if the operation is not addition)

While a class member is out of the room, the rest of the class decides upon a problem for the missing class member to solve. Claps represent a number; snap of fingers represents where the sign is to be used (depending upon operation used.) In multiplication CLAP-CLAP-SNAP-CLAP-CLAP-CLAP would represent the problem 2x3=6. If it were addition, it would represent 2+3=5. Member being challenged claps back the answer -- or names it (whatever you decide.)

Each numeral is represented by the number of claps it represents. The number 5 would require 5 claps, etc.

DIGIT DISPERSION (Place Value Game)

Use two sets of number cards which have the numerals 0 through 9 from which to draw; or, use a spinner to randomly select numbers. Have child draw boxes on a paper to represent the type of problem to be calculated. When randomly selected numeral is called, student places it in a box and cannot erase it. Upon completion of the digit selection, student performs operation. The person who receives largest (or smallest answer -- whichever was designated before the numeral selections began) is the winner.

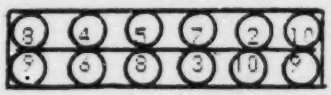


EGG CARTON PRACTICE
Materials: 2 chips and an egg carton

Write a number in each of the 12 spaces in an egg carton (may repeat the number). It is best to choose the larger numbers, to be placed in any order. Child closes lid, shakes chips, and reads the two numbers wherein the chips lie. He may give the sum, difference, product, or quotient -- whatever has been designated as the operation by the teacher.

Students may work in pairs and check each other. Teacher may choose to shake the carton and check an individual's facts.

M. A. Harner.



COLD CALCULATION

Ice cube trays can be filled with mini flash cards to be placed in proper cups which have been labeled with numbers. A self-checking list is taped to the ice cube tray. When a child can say all of the facts of a given operation, he/she becomes a member of the Math Club and his photograph is placed on the bulletin board.

More motivation can be brought about by competition of boys vs. girls.

Charlene Harlan

PEEK 'N POKE GAME (Game for 1 or 2)

Use tag board, preferably in gay colors. Cut in any shape: rectangle, ghost shape, turkey shape, Santa shape, etc. With paper punch, punch holes all around the shape. Above the holes write the math fact on one side and the answer on the other. Child places pencil in hole, says the fact, and can check his answer immediately, or his pal can check by looking on reverse side. (Can cover with contact paper or laminate, if desired.)

Front side

2	3	5	6	3	6	8
+2	+4	+5	-1	x2	x3	-4

Back side

4	7	10	5	6	18	4
•	•	•	•	•	•	•

Joan Locher

MULTIPLICATION BINGO

Prepare bingo cards with products in the squares. The caller calls the facts, and players cover the answers (products). Use chips, corn, or whatever. Caller must keep cards called so she can check to see if all products are represented in the cards called.

Sharon Veale

GIANT STEP FOR MATH-KIND

Make large footprints out of brown paper. Place difficult math facts on them. Select a part of the room and call it moon crater. A pupil's moon walk will be successful if he/she can get all facts correct when using the footprints to cross the moon crater.

9x6

4x9

6x7

SLIDE RULE FOR ADDITION AND SUBTRACTION PRACTICE

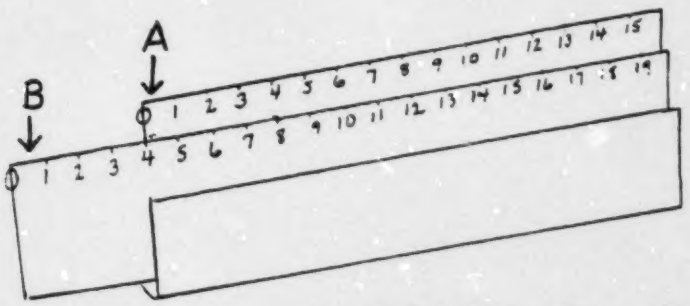
DIRECTIONS FOR MAKING:

1. Cut a rectangle from oaktag about 9 1/2 inches wide and 3 inches high.
2. Cut another rectangle from oaktag 9 1/2 inches wide and 5 1/2 inches high.

   a. Place vertically and measure up 1 3/4 inches from the bottom.
   b. Draw a line across this 1 3/4 measurement and fold the oaktag card along the line.

3. Divide each piece of oaktag into equal units along the top vertical and number from 0 as far as you wish to go. (Size of numbers and distance between them will depend upon grade level.)

   a. Be sure the spacing and numbers are identical, matching exactly, on each piece of oaktag.
   b. Use different color marker on each piece.



Insert B into A. For primary grades you could tape Oaktag A to the desk.

DIRECTIONS FOR USAGE:

SUBTRACTION AS SHOWN ON THE ILLUSTRATION FOR 7 - 4 = 3

Keep in mind that Sum minus Addend = Addend:

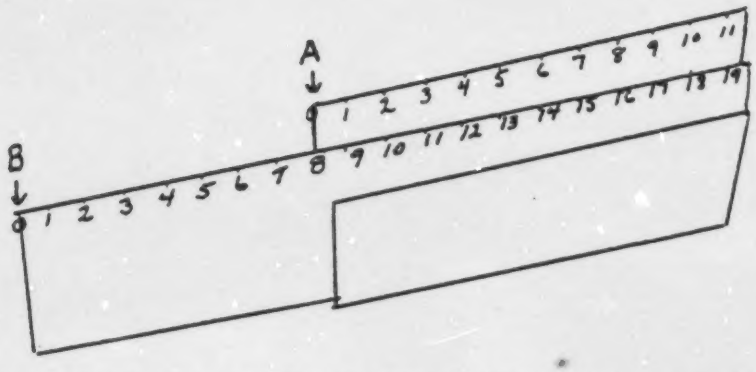
a. Slide Oaktag B until the 4 (or the Addend to subtract) on Oaktag B is below the 0 on Oaktag A.

b. Now look for the Sum 7 (or whatever sum a student is subtracting a number from) on Oaktag B.

c. Directly above the 7 on the illustration above is the other Addend 3, which is the answer to the 7 - 4.

The addend which is the answer to the subtraction problem will appear on Oaktag A, directly above the Sum of the problem on Oaktag B.

The addend being subtracted will appear on Oaktag B, directly below zero on Oaktag A.



ADDITION ON A SLIDE RULE AS SHOWN ABOVE FOR 8 + 6 = 14

Keep in mind that Addend + Addend = Sum

a. Slide Oaktag B to the left so that addend 8 is directly beneath the zero on Oaktag A.

b. Find Addend 6 on Oaktag A.

c. Directly under 6 on Oaktag A is 14 on Oaktag B. 14 is the Sum of 8+6.

EQUALS CHANGE

Prepare large numeral cards and have them on strings so they may be hung from a child's neck.

Children stand in a circle, with a number card around their necks.

These numbers will represent the answers to facts being studied. (If addition facts through the fives are being studied, you will need answer cards through ten, etc.)

One student is designated as "NO NUMBER" but remains in the circle with the other NUMBERS and wears nothing around the neck.

Teacher calls out two numbers:	Action resulting for examples below:
a. 2 addends, if addition	a. 5+3 (2 students wearing number 8 go to center of circle.)
b. 2 factors, if multiplication	b. 3x3 (2 students wearing number 9 go to center of circle.)
c. a sum and an addend, if subtraction	c. 9-6 (2 students wearing number 3 go to center of circle.

d. a product and a factor, if division	d. 8 2 (2 students wearing number 4 go to center of circle.)

If the student designated as "NO NUMBER" runs to the center of the circle and gives the correct answer before the student wearing the correct answer reaches the center of the circle, he becomes the number worn by the student whom he has beaten, and the student he has beaten becomes "NO NUMBER".

WHO CAN?

Have every child make a set of numbers which will be suitable answers to facts being learned. Teacher holds up a number and poses a question. Children find answer in their cards and hold up their answers. When all of the students have responded with a card, the teacher holds up the card that is the answer and all repeat the fact in unison. This is immediate feedback for teacher and child, and nobody has to be reminded he is wrong.

Examples: Teacher: "I have a 5 (holds it up). Who can make it 13?"
Student: Holds up an 8. (All recite the fact together -- 5 + 8 = 13).

Teacher: "I have an 8 (holds it up). Who can make it 56?"
Student: Holds up a 7. (All recite the fact together -- 8 x 7 = 56).

NUMBER STOMP

Pupil goes to the front of the room and stomps out a number, bows for the sign, and stomps out his other number. The bow can mean +, -, -, or x, depending upon the operation being used that day. Pupil only does the stomping once -- will not repeat. The person giving the complete fact first gets to do the stomping.

OR --

You can form teams and the opposite team must try to answer the problem stomped out. If the first team member misses, go on to the next team member. If he misses, go on to the next. Repeat this until you get a correct answer. The team doing the acting gets a point for every student who misses. A penalty point will be given if anybody whispers the answer.

BUZZ (multiplication)

Whole group stands, preferably in a circle. The teacher chooses the number to be "BUZZ". For example, the teacher may choose the number 4 to "BUZZ". Children count off around the room, or circle. Any number with a four in it, or, is a multiple of 4 is a "BUZZ". If a child says the number instead of "BUZZ", he or she sits down and the play continues until one person is standing. Teacher may have children start over with a 1 or take up where student who missed left off.

Example: 1, 2, 3, "BUZZ", 5, 6, 7, "BUZZ", 9, 10, 11, "BUZZ", 13, "BUZZ", 15, "BUZZ", 17, 18, 19, "BUZZ", 21, 22, 23, "BUZZ", etc.

Barbara Chilton

THE NINE'S FACTS ON THE FINGERS

Below are sets of fingers showing how to find the nine's number facts easily.

Note in the drawings, that the finger down is the number of times you wish to multiply by nine.

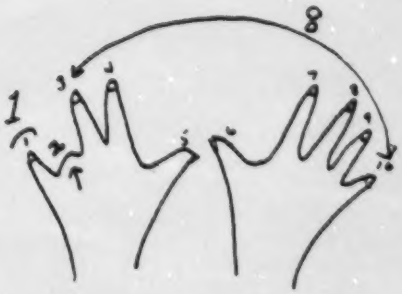
Count the fingers to the left of the one down and you have the number that belongs in the ten's place of your product.

Count the fingers to the right of the one down and you have the number that belongs in the one's place of your product.

1X9=09

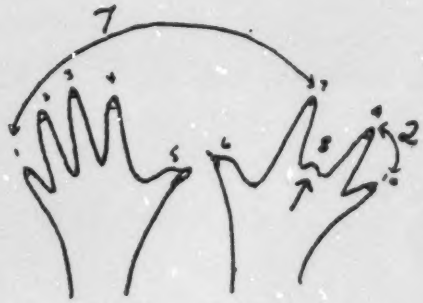
4x9=36

7X9=63

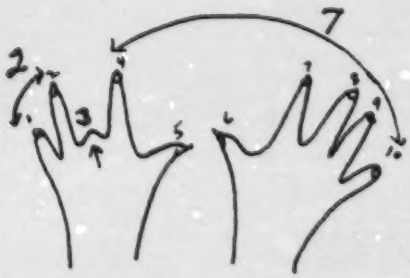


2X9=18

5X9=45

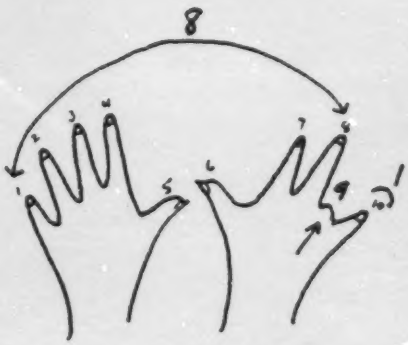


8X9=72



3X9=27

6X9=54



9X9=81

SONG TO HELP WITH LONG DIVISION (Tune of Davy Crockett)
Submitted by Flossie Hulsizer

First you divide
And then you multiply;
Draw a line and then subtract.
Make that remainder mighty small --
Compare with the divisor, and that's not all.

Chorus:

Long, long division
Headache of the whole fourth grade.

Bring the next number
Down by the side.
Now start over and divide
Multiply, subtract, and then compare
If there's a remainder you put it right up there.

Chorus:

(repeat)

Note:

When using this song, be sure you have problem examples on the board and show the student the meaning of the song as you walk through an examole.

The child must understand what he/she brings down and to where he/she brings it down. What is "the side?" By the side of what?

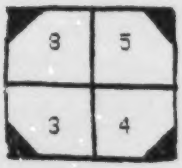
This song is a reinforcer of the steps and should be used after the algorithm has been worked through and understanding is established. It won't do the teaching for you.

Where is "right up there" that you are going to place the remainder?

PATTERN MATH

Use page 123 to duplicate for students.

Give them numbers to place within the small squares in the large squares above. Perform the operation indicated by the teacher on the numbers which the pattern represents.



If we are adding:	8+5+4=17	8+4=12	3+4=7	8+5=13	5+4=9	5+3=8
If we are multiplying:	8x5x4=160	8x4=32	3x4=12	8x5=40	5x4=20	5x3=15
If we are subtracting:	8-5-4= -1	8-4=4	4-3=1	8-5=3	5-4=1	5-3=2
		4-8= -4	3-4= -1	5-8= -3	4-5= -1	3-5= -2

If we are dividing: $\frac{8}{5} \div 4 = \frac{2}{5}$ $4 \div 8 = \frac{1}{2}$ $3 \div 4 = \frac{3}{4}$ $5 \div 8 = \frac{5}{8}$

$8 \div 4 = 2$ $4 \div 3 = 1\frac{1}{3}$ $8 \div 5 = 1\frac{3}{5}$ $4 \div 5 = \frac{4}{5}$ $3 \div 5 = \frac{3}{5}$

$5 \div 4 = 1\frac{1}{4}$ $5 \div 3 = 1\frac{2}{3}$

WORKSHEET FOR ADDING TWO-PLACE NUMBERS, pg. 124.

POP OVER (for any operation)

(Two students play and one person, who has instant recall of facts being studied,. Keeps score.)

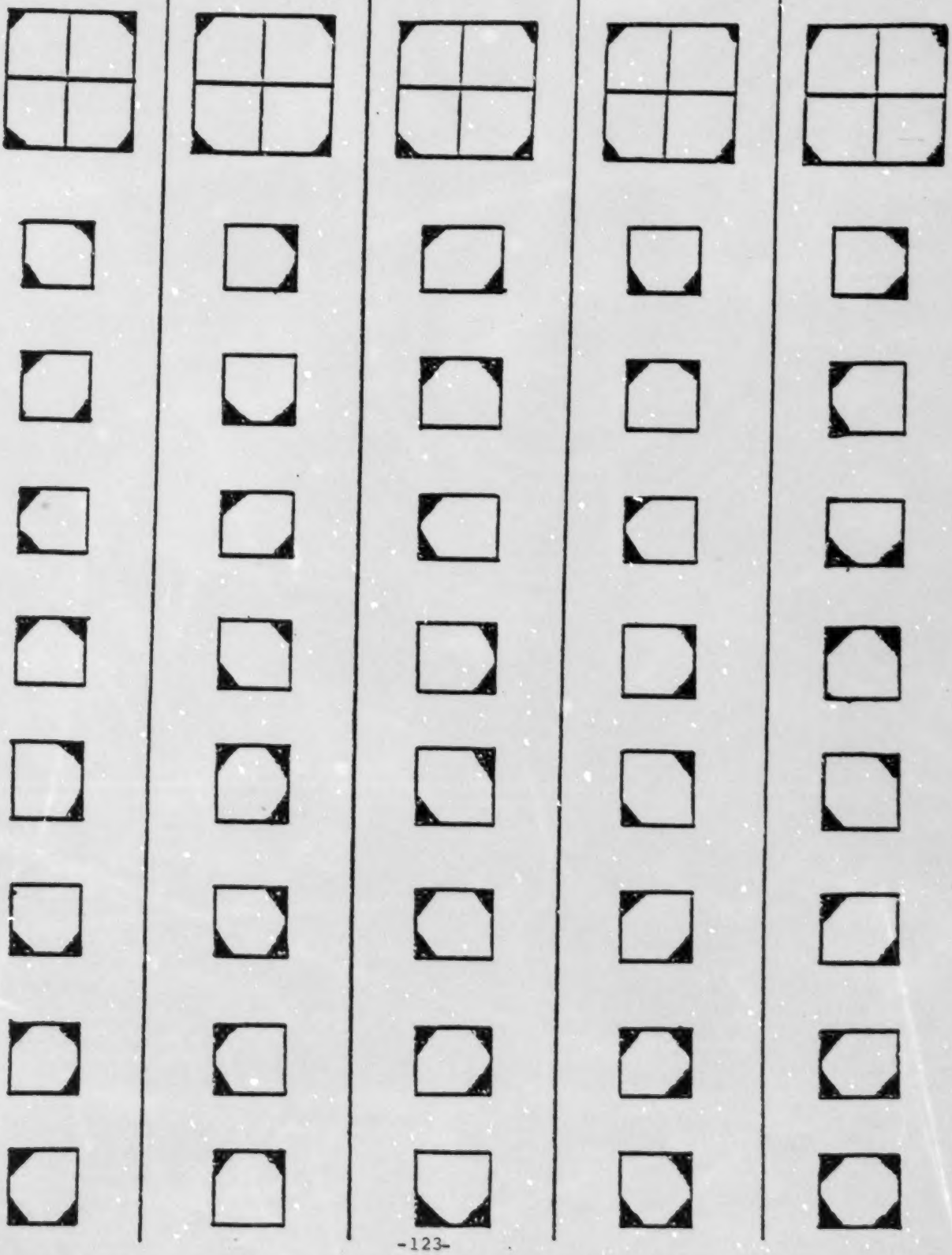
Use all the cards between 2 and 10 from an old deck of cards. Decide rules for the game before first card is turned over. Set game winning score.

First student picks up a card and says, "My card is:

Examples: 5 more than 4
2 less than 6
4 greater than 2
3 times 3

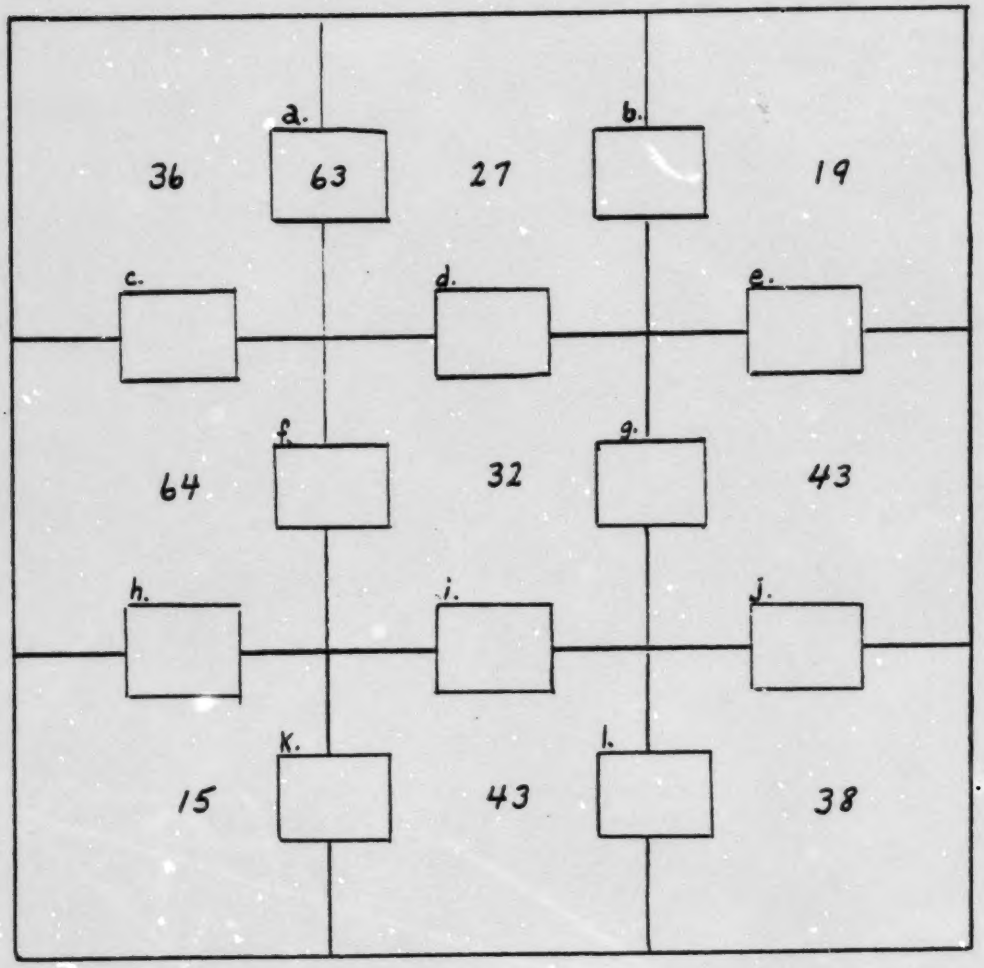
Other player responds with answer to what the first player wants. If the player gives a wrong clue, deduct a point. If the player answering is wrong, deduct a point. If the player is correct, add a point to score.

(Pattern Math to be used with page 122)



WORKSHEET FOR ADDING TWO-PLACE NUMBERS

Directions: In each ☐, write the sum of the numbers in the two adjoining boxes. One answer has been done for you.



1981, by the Center for Applied Research in Education, Inc.

ADDEND SPIN (facts through 18)

Choose teams with any number of players. Use a spinner with digits 0 through 9. Player spins and must add the number on which arrow lands to all the other numbers on the spinner. Player must not hesitate or be given a second chance to give the correct sum. If player is correct, the team adds a point to its score.

Set time limit or winning score before game begins.

CALENDAR ARITHMETIC

Add the numbers in each row.
Add the numbers in each column.
Add the numbers in the diagonals.
Add the numbers in every other square.
Add the even numbers.
Add the odd numbers.

A. Select a three-by-three square. Add the numbers in each of the diagonals. (The sums will be the same.) Add the numbers in the center column and find the sum of the numbers in the center row. (All four sums will be the same.)

B. Select a four-by-four number square. First find the sums of the diagonals. Find the sums for the first three columns, starting at the left. Then find the sums for the first three rows, starting at the top.

S	M	T	W	T	F	S
	1	2	3	4	5	6
7	8	9	10	11	12	13
14	15	16	17	18	19	20
21	22	23	24	25	26	27
28	29	30	31			

Study the sums for the first three columns and predict the sum of the fourth column without adding. Do the same for the fourth row.

Have pupils try to figure out why the sum for each row in the 4 by 4 square is 28 larger than the sum for the row above it, and why the sum for a column is four larger than the column to its right.

DOLLAR WORDS

Assign a value to the letters of the alphabet and have students find the value of words. (a=1¢, b=2¢, c=3¢, etc.) This can be done with the spelling words, reading words, or lists of words made up by the teacher. Have a contest to see who can find the most dollar words.

(Refer to page 136 for page of words to duplicate.)

PLACE VALUE GAMBLING (practice 3-digit addition) (Obj. 18 pg. 16)

Teacher draws from a set of cards numbered 1 through 9. Each child has a grid as shown. Each player places the number on one of the six spots on his playing grid, or on the space for one he wishes to throw away. The teacher calls out 7 numbers. Once a digit is placed on a space, it cannot be moved. Winner is the player with the highest sum for his six numbers.

Castaway

_____

_____ _____ _____

\+ _____ _____ _____

JUST EIGHTEEN (addition facts to 18) 2 players

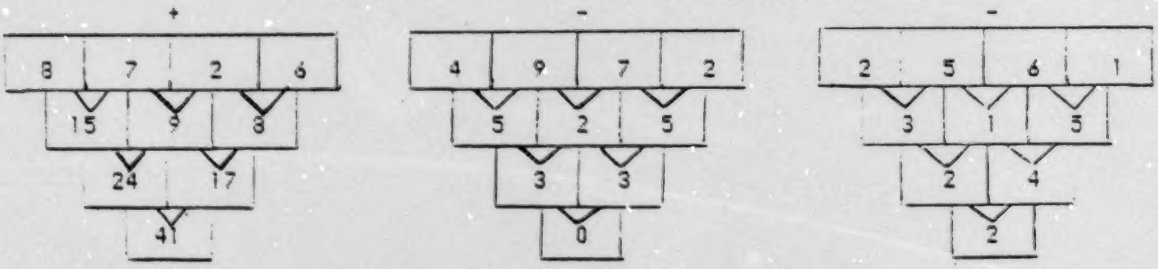
Deck of playing cards with Aces and Queens removed. Jacks = 12 and Kings = 13. The rest of the cards have their face value. Set winning score or time limit on game.

Each player gets seven cards. The rest are placed face down on the table. The first player lays a card down next to the stock pile and calls out its value. The next player plays a card on top of it, hopefully making a sum of 18. If not, the first player adds to find the sum of the two cards played and sees if he/she can put a card down and bring the total to 18. If player can, he/she may pick up the cards whose sum is 18.

If a player cannot make a play without going over 18, he/she must pick from the stock pile until a play can be made. The first player to get rid of all cards from his/her hand is the winner. Winner's score is the sum of the cards in opponent's hand.

REVERSE PYRAMIDS (+ or - practice)

Place any four numbers in the top boxes.



SIX-DIGIT RELAY (6-member teams are set up)

Using six-digit numbers, the teacher reads a four or five number addition problem for first student to write. First student adds the ones and records answer. Second student on that team adds the tens, and writes his answer, etc. until all teams finish the problem. Have them place the order of finishing above their problem. First to finish correctly gets 6 points; second, 5 points, etc. If a team's answer is wrong it subtracts a point. Here is a good place to introduce negative numbers. Team with most points wins the game. Be sure that teams are evenly grouped according to abilities.

Ex. 234567
765432
879645
546978

or

	2	2	2	2	2	
	2	3	4	5	6	7
	7	6	5	4	3	2
	8	7	9	6	4	5
+	5	4	6	9	7	8
2,	4	2	6,	6	2	2

22 = sum of ones
200 = sum of tens
2400 = sum of hundreds
24000 = sum of thousands
200000 = sum of ten-thousands
2200000 = sum of hundred-thousands
2426622 = sum of all columns = 2,426,622

ADDITION AIDS

PALINDROMIC NUMBERS

Palindromes read the same in reverse as they do forward. e.g., 383, 2332, 21812, etc.

Begin with any number. Reverse its digits and add the two numbers, e.g., 142+241=383. You get a palindrome on the first reversal. If, however, you do not get a palindrome on the first reversal, repeat the process until a sum is reached which can be read the same from either end (said to be palindromic.)

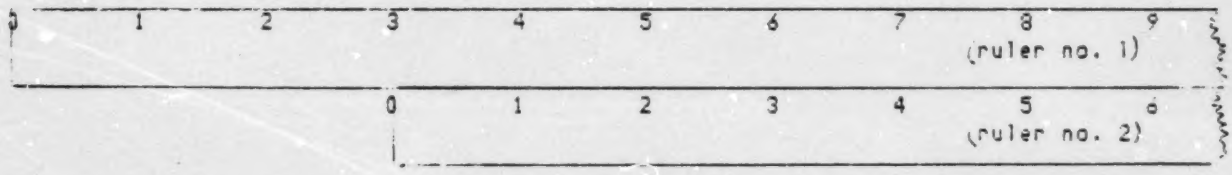
Children can record the number of steps needed to reach a palindrome and classify the numbers they began with as one-step, two-step, three-step, etc.

Several examples are shown:

158 + 851 = 1009
1009 + 9001 = 10010
10010 + 01001 = 11011 (palindromic)
(3-step)

174 + 471 = 645
645 + 546 = 1191
1191 + 1911 = 3102
3102 + 2013 = 5115 (palindromic)
(4-step)

SLIDE RULE ADDITION AND SUBTRACTION (Obj. 6, pg.13) (See page 116 for patterns.)



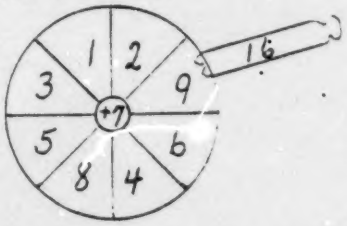
Two rulers may be used as a device for adding or subtracting numbers.

If one wishes to add 5 to 3, place ruler no. 1 on top. Then place 0 on the second ruler directly below the 3. Then follow ruler no. 2 down five spaces. Looking immediately above the 5 on the second ruler, find the number 8 on ruler no. 1. This shows that 3 + 5 = 8.

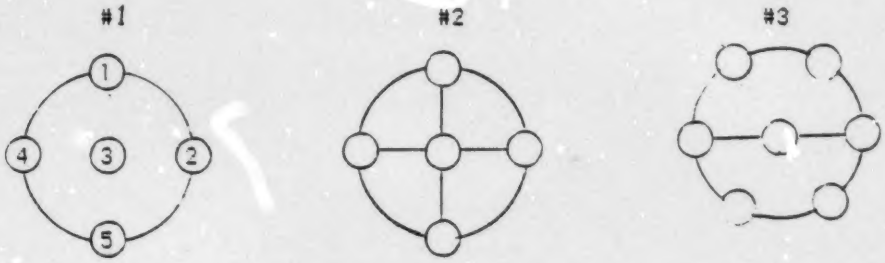
For subtraction, reverse the procedure. To subtract 5 from 8, place 5 on ruler no. 2 directly under 8 on ruler no. 1. The zero on ruler no. 2 will be directly under the difference when you subtract 5 from 8.

FACT PLATES

Make plates that have numerals on them. Using clothespins with numerals on the end, clip them beside the number that is the answer to the operation specified in the center of the plate.



MERRY-GO-ROUND



Objective: To provide practice in addition and experience in finding patterns.

Directions for teachers:

1. Prepare an activity sheet with the diagrams shown above.

2. The first merry-go-round is completed. The first five counting numbers were placed in the five circles so that the sum of every three numbers in a straight line is 9.

3. Use merry-go-round no. 2 for these activities. Try to place the five numbers given below in the five circles so that the sum of every three numbers in a straight line is the given sum. You must use each of the five numbers, and each number may be used once and only once.

   a. The first five even counting numbers. Given sum is 18.
   b. The first five odd counting numbers. Given sum is 15.
   c. Any five consecutive counting numbers. e.g., 5, 6, 7, 8, 9. Given sum is 21.

4. Use merry-go-round no. 3 for these activities. The directions are the same as above, except now there are seven numbers to place.

   a. The first seven counting numbers. Given sum is 12.
   b. The first seven even counting numbers. Given sum is 24.
   c. The first seven odd counting numbers. Given sum is 21.
   d. Any seven consecutive counting numbers, e.g., 7, 8, 9, 10, 11, 12, 13. Given sum is 30.

QUESTIONS:

1. Did you notice any pattern?
2. Do you notice anything special about the number that is placed in the center of the circle of the merry-go-round?
3. What do you notice about the numbers paired with the one in the center of the merry-go-round?

Source: Sister Mary Petronia Van Straten, SSND, "For Your Classroom Elementary", in Wisconsin Teacher of Mathematics, Vol. XXVIII, Number 3, Fall, 1977, pp. 4-6.

ADDITION PRACTICE

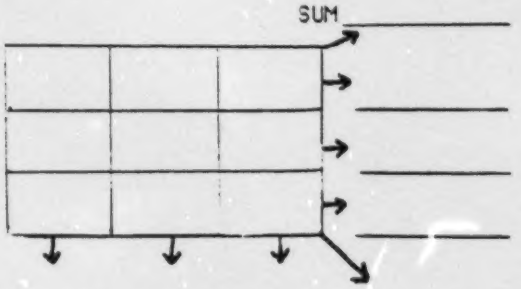
SUM OF SAME SUMS (any number of players)

Each player makes a grid, three-by-three, or four-by-four, as designated by the teacher.

Teacher can roll a die, use a spinner, or whatever, to randomly select the numbers to be used. When the teacher calls out the number, the student places it on grid wherever he/she chooses. Once it is placed, the number may not be moved to another space.

The teacher calls out numbers until all spaces on the grid have been filled.

Players add each row, each column, and the two diagonals on their grids. Next, each player circles the sums that appear more than once on his/her grid. The player adds the circled sums. This is his/her score. The player with the largest score is the winner.



SUMS APPEARING MORE ONCE

SCORE _____

SUBTRACTION PRACTICE

DIFFY (A Self-generated drill)

Student places any numbers he/she wishes in the four corners of the game sheet.

Next, pupil subtracts the smaller one from the larger one in the opposite corner circle and places answer in the middle circle. Thus, there will be a number in every middle circle.

Then, subtract each of those opposite circles and place answer between the two numbers used in the subtraction process.

Pupil continues this procedure until he/she arrives at zero. That ends the game.

Variation can be done in a vertical way.

Explanation of charting shown:

The number placed on the extreme right of each row is obtained by subtracting the smaller from the larger number of the numbers at the beginning and end of each row.

The 3 at the right of the second row is obtained by subtracting 2 from 5 in the first row.

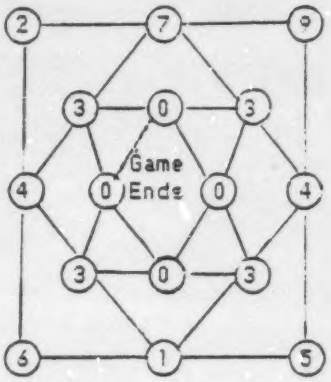
The 4 at the right of the third row is obtained by subtracting 3 from 7 in the second row.

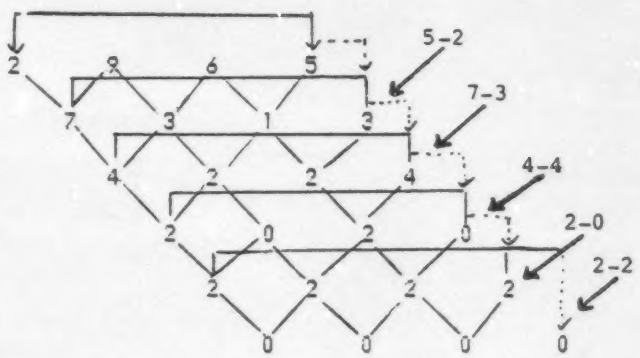
The 0 at the right of the fourth row is obtained by subtracting 4 from 4 in the third row, etc.

Keep subtracting until you come to zero.

Remember, you always subtract the smaller from the larger number.

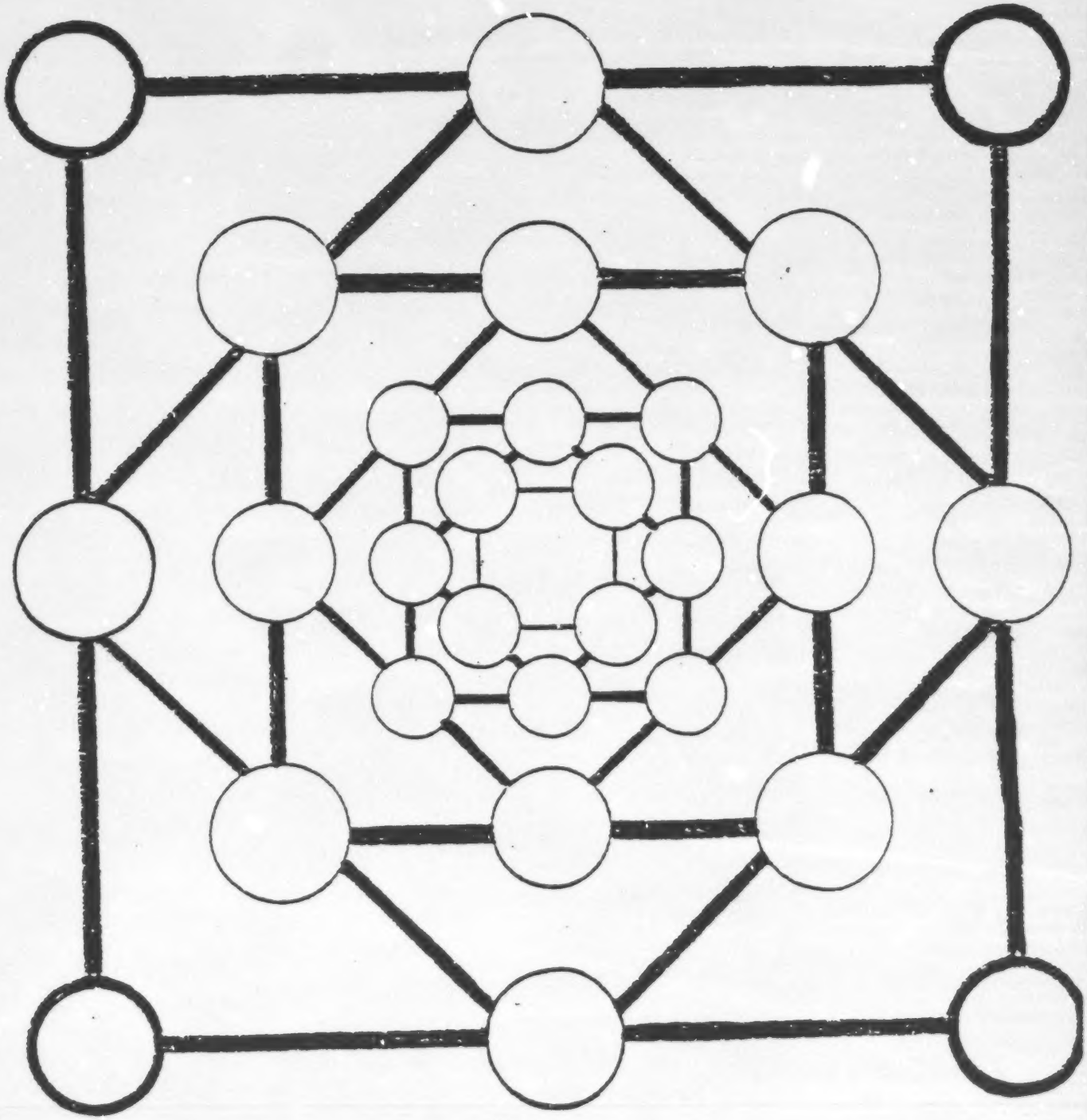
(Page 132 to be duplicated for Diffy.)





Older students may try fractions!

WHAT'S THE DIFFERENCE
SUBTRACTION DRILL



TEACHER IDEAS

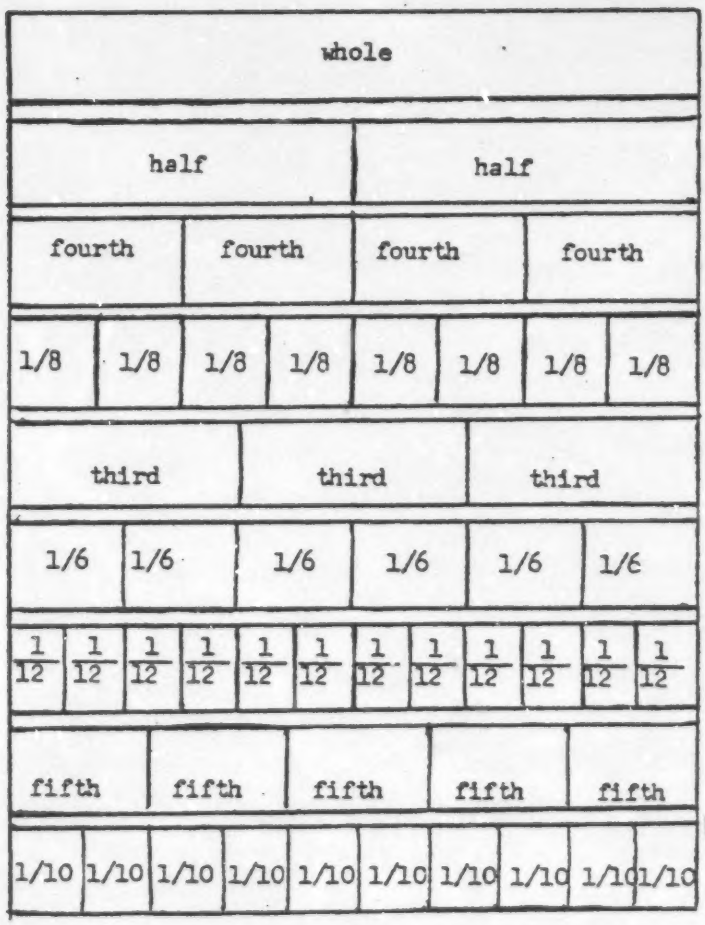
OBJECTIVE	PROCEDURE	ACTIVITY
1. To identify numerals 0 - 10	1. 0-10 number cards are mixed and shown to class (sitting on floor) with a target number named. When target number appears, class does directed* activity. As each number is shown class (or individual) names it twice. Mix and repeat.	1. Example of directed activity: "Today jump 8 times on Number 8; or, Hop 6 times on Number 6; or Clap, Tap your feet, Pat your head, or Turn around, etc."
	Bonnie Yeager, Kdg.	
2. To recognize the symbols $>$ greater than and $<$ less than.	2. Use the Pac Man craze to help children realize which way the the open part of the symbol should go. Pac Man wants to eat the larger number. It worked ! Myra P. Cooke	
3. Recognizing and reading Numerals 0-12.	3. Count objects in the room. Draw sets of a given number of objects.	3.a.Hap Palmer Vol. I Number March (1-5) b. Math fingerplays c. Children make number books.
	Resources: The Handbook of Learning Activities for Young Children, by J. Caballero; Sew and Know--Skill Based Puppet Projects to Teach 1-12. The Monkey Sisters.	
	Mary Mehoff	
4. To teach the making of numerals one through nine.	4. Song to the tune of "Here We Go Round the Mulberry Bush."	4. Make each numeral three times.
	(1) Pull straight down and that is all; Pull straight down and that is all; Pull straight down and that is all To make a number 1. (2) Curve, slant and then you slide, etc. (3) Curve around and curve again, etc. (4) Pull, slide, and then you cross, etc. (5) Pull, curve and add a hat, etc. (6) Slant down and make a loop, etc. (7) Slide, slant and that is all, etc. (8) Curve, curve and back again, etc. (9) Around, close, and pull to the line, etc.	
	Pat Broderick	

OBJECTIVE	PROCEDURE	ACTIVITIES
5. Rounding of numbers by the boxing and circling method.	5. Box off the place values involved in your rounding. Ex. Using the number 2385: I. a. to round off to tens, box in the ones and tens places: 2 3 [8 5] b. To round off to hundreds, box in the ones, tens and hundreds: 2 [3 8 5] c. To round off to thousands, box in the ones, tens, hundreds, and thousands: [2 3 8 5] II. Next step is to circle the first two numbers in the left side of the boxed in numbers: a. 2 3 [(8 5)] b. 2 [(3 8) 5] c. [(2 3) 8 5] III. Step number three is to round off the two numbers circled. RULE: If there is a number in the ones place of the number you circled that is 5 or greater, you round up the number to its left. If the number in the ones place of the circled number is less than 5, you round down the number to its left. 9 0 a. 2 3 [(8 5)] = 2390 8 0 4 0 0 b. 2 [(3 8) 5] = 2400 3 0 0 3 0 0 0 c. [(2 3) 8 5] = 2000 2 0 0 0	

Ann Greer

From "Fraction Bars Kit" by Albert B. Bennett, Jr., and Patricia S. Davidson (Creative Publications, Palo Alto, CA).

Referred to under Objective 1e. page 57.



DOLLAR WORDS

a	b	c	d	e	f	g	h	i	j	k	l	m . . .
1¢	2¢	3¢	4¢	5¢	6¢	7¢	8¢	9¢	10¢	11¢	12¢	13¢ . . .

analysis	appointed	asbestos	attitude
babybuggies	bedposts	borrowed	carpenter
choppers	congress	crackerbox	delivery
digestive	discipline	ducklings	elephants
excavates	excellent	fountain	gladiolus
gypsies	honeycomb	hospital	irritate
judicary	keyholes	kleenexes	knothole
lacquerer	lampstand	landscaping	lightning
mailboxes	merrily	mirrored	motors
ninepins	performed	personal	pistles
powders	practicing	prevent	primary
problems	pumpkin	quarter	refinery
revives	resolved	roomful	sagebrush
services	session	shadowing	shoots
shovels	squares	standards	status
stoves	straws	swimmer	styles
tabernacles	telephone	thirty	threatened
toilets	towards	towers	traction
turkey	variety	violins	weaving
Wednesday	whirling	whisky	whistled
wholesale	wizard	writing	

TEACHER REFERENCE MATERIALS

BOOKS IN ELEMENTARY SCHOOL LIBRARIES:

Magic House of Numbers, Adler
Wonders of Mathematics, Feravolo
Wonderful World of Mathematics, Hogben
New Ways in Mathematics, Jonas
Learn to Fold, Fold to Learn, Abbott
Mirror Magic, Abbott
Making and Using Graphs and Nomagraphs, Denholm
Learning About Measurement, Horne
Patterns and Puzzles in Math, Horne
Arithmetic Can Be Fun, Leaf
Paper and Pencil Geometry, Roper
Easy Book of Fractions, Whitney
From Fingers to Computers, Willerding
Math Around the Clock, Willerding
Names, Sets and Numbers, Bendick
Roman Numeral, Adler
Shapes, Sides, Curves and Corners, Podendorf
Circles, Triangles, and Squares, Hoban
Sesame Street Book of Shapes

AIDS IN SCHOOL LIBRARIES:

Math Balance
Cuisenaire Rod
Flash cards
Fraction Circle Kit
Fraction Line Set
Kit -- Modern Math Made Meaningful, Kenwan

ON LOAN FROM EMC:

Using Consumer Math -- kit
Math Map Trip
Math for Young Consumer, SF 4394 T
Learning With Rods, 4385 T
Introducing Graphs, SFS 4257 T
Living with Computer, SFS 4377 T
Introducing Geometry, SFS 4345 T
What Is a Fraction, SFS 4347 T
Introducing Addition, Subtraction, Multiplication, Division, SFS 4346 T
Basic Colors, SFS 4321 T
Basic Shapes, SFS 4322 T
Telling Time, SFK 4240 TP

SOITA TAPES (see book at your school):

Math No Mystery
It Figures
Measure Up
Math Relationships

What are Fractions and How They Are Used, film, MP 1594 T
What Is a Fraction? SFS 4347 T
Parts of Things, film, AA 0249 E
Fractions II, Some Operations With Fractions, MP 1595 T

I strongly urge every school library to:

1. Subscribe to Arithmetic Teacher, National Council of Teachers of Math, 1906 Association Drive, Reston, Virginia,

2. Obtain the two publications about problem solving and one titled Computation, or Changing Perspective, from the Ohio Department of Education, Office of Inservice Education, 65 S. Front Street, Columbus, Ohio 43215.